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OPTIMIZATION MODELS FOR UNDERWAY REPLENISHMENT OF A DISPERSED CARRIER BATTLE GROUP

by

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March 1992

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Optimization Models for Underway Replenishment of a
Dispersed Carrier Battle Group

by

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Submitted in partial fulfillment
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ABSTRACT

This thesis presents a classification of basic optimization models for planning underway replenishment of a battle group. In particular, this thesis focuses on two scenarios, *routine* and *rearming*, and considers three replenishment tactics: circuit rider, delivery boy and gas station. Some of the models presented can be classified as a (standard) traveling salesman, generalized traveling salesman or orienteering problem. However, several models are further generalizations of these problems which have not been previously considered. Computational experiments using four formations from the literature and commercially available software identify problems that are difficult to solve and/or require specialized algorithms.

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The reader [illegible]tioned that computer programs developed in this research may not ha[illegible] exercised for all cases of interest. While every effort has been made, with[illegible]me available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

Carrier battle groups have the unique capability to deploy military striking power anywhere in the world. They can operate in international water without relying upon the support or cooperation of other governments. This capability is essential to carrying out the strategy of forward engagement, i.e., the strategy of waging war in the enemy's backyard instead of our own. During peace time, the mobility and flexibility of carrier battle groups also provide the ability to project relatively unobtrusive 'over the horizon' presence which is indispensable for responding to and, perhaps deterring, crises and low intensity conflicts around the globe.

One of the most important factors in insuring the effectiveness and survivability of carrier battle groups is sustainability. Battle groups must be capable of carrying the fight to the enemy as well as sustaining combat operations for an extended period of time. Certainly, the degree of sustainability depends on the level of logistic support. One way to insure a high level of such support involves a three-step process. The first step is to locate 'advanced logistics support bases' (ALBSs) close to areas of potential conflict. In the second step, supplies and materiel are transported from ALBSs to battle groups by 'shuttle' ships. Finally, assets such as station ships and helicopters distribute the needed supplies and materiel to other ships in the

battle group. The distribution of supply and materiel by a station ship is often referred to as 'underway replenishment' or simply 'unrep'. To avoid confusion, this thesis refers to a station ship as a supply ship.

The tactical disposition of ships in a carrier battle group has changed over the last forty years due to the increasing sophistication of naval technology. In the past, a formation of ships was usually extended over a few thousand yards. Today, a modern battle group is typically dispersed over a very large geographical area. This large dispersement of ships makes the task of developing efficient unrep plans more complex. Two similar, but different, replenishment sequences can take a drastically different amount of time to complete due to the large distances ships must traverse. Thus, the current method of manually scheduling replenishment may no longer be efficient, for it may lead to an unnecessarily long replenishment time. The objective of this thesis is to facilitate the development of effective and efficient unrep plans. In particular, this thesis models unrep scheduling as optimization problems. Under varied tactics and scenarios, some models reduce to well known problems in routing and scheduling while others represent new generalizations not previously explored. Thus, the results in this thesis not only point out new applications, they also enrich a well known class of problems in routing and scheduling.

The outline of this thesis is as follows: Chapter II reviews the existing literature on underway replenishment and provides basic background on

traveling salesman problems. Chapter III provides optimization models for planning unrep using three tactics and two operational scenarios. These models were implemented and the resulting problems were solved using commercially available software. Computational results are reported in Chapter IV.

II. BACKGROUND

In this chapter, the first section describes the basic components in unrep planning. To put this type of planning in perspective, a description of the traveling salesman problem (TSP) is given in the following section. Finally, the last section reviews prior work in unrep planning.

A. UNDERWAY REPLENISHMENT PLANNING

In the simplest terms, unrep planning is specifying the sequence and locations for combatant ships to be replenished. In practice, unrep planning involves synchronization of many facets of carrier battle group operations. In order to define and characterize underlying analytic models, major components of the unrep process need to be identified and categorized. Table 1 summarizes the basic features of unrep planning. Some are self-evident and others are explained below.

TYPE OF UNREP: There are two basic methods for transferring supplies and materiel between two ships at sea. One method is called vertical replenishment or VERTREP. Here, a logistic helicopter is used to lift pallets or containers from a supply ship to a receiving ship. For the other method, connected replenishment or CONREP, the supply ship travels alongside the receiving ship and the delivery of supplies is accomplished by means of cables,

e.g.,"high lines" rigged between them. The models in this thesis address the latter form of unrep.

TYPE OF ASSETS: Helicopters are the only assets for VERTREP. For CONREP, two types of vessel are typically used in the role of supply ship: fast combat support ships (AOEs) and fleet replenishment oiler (AORs). However, an AOR is usually supplemented by an ammunition ship (AE) and combinations of AO and AE supply ships are also employed.

UNREP TACTICS: Hardgrave[1989] listed four replenishment tactics for unrep: gas station, delivery boy, circuit rider and chain saw. Only the first three are considered in this thesis. The last tactic can be analyzed in a manner

TABLE 1. BASIC FEATURES OF UNREP PROBLEMS

Features	Descriptions
Type of unreps	Vertical replenishment (VERTREP) Connected replenishment (CONREP)
Type of assets	Helicopters, Supply ships (AOEs, AORs)
UNREP tactics	Delivery Boy, Circuit Rider, Gas Station
Number of simultaneous unrep	unrep one ship at a time, unrep several ships at a time
Objective function (for CONREP)	Minimize time to unrep, Maximize combat value
Other	Restriction on the no. of ships "off-station", Speeds of formation and supply ship, Time period in which a combatant ship can be replenished.

similar to the other three; however, its advantages in battle group defense is subject to much debate.

In the gas station tactic, the supply ship remains at its position inside the battle group formation. When a combatant ship is scheduled to be replenished, it leaves its position in the formation and comes alongside the supply ship. On the other hand, in the delivery boy tactic, the supply ship travels around the formation and visits combatant ships requiring replenishment in some specified sequence. So, in gas station, the supply ship remains in its position and combatant ships move away from their positions to perform unrep and the reverse is true for delivery boy. The third tactic, circuit rider, is a hybrid or a compromise between the first two, in that both the supply ship and combatant ships move away from their positions to some specified rendezvous locations to perform unrep. For this third tactic, the best rendezvous location would depend on the objective or goal to be achieved. Note that when all rendezvous locations are the same as the position of the supply ship, circuit rider becomes gas station. When rendezvous locations are set to the positions of combatant ships, circuit rider is identical to delivery boy. Figure 1 graphically illustrates these three tactics.

NUMBER OF SIMULTANEOUS UNREP: This feature can apply only to gas station and circuit rider since the delivery boy tactic dictates that the supply ship visits each combatant ship in some sequence, hence the supply ship can only unrep one ship at-a-time. However, for the other two tactics, the

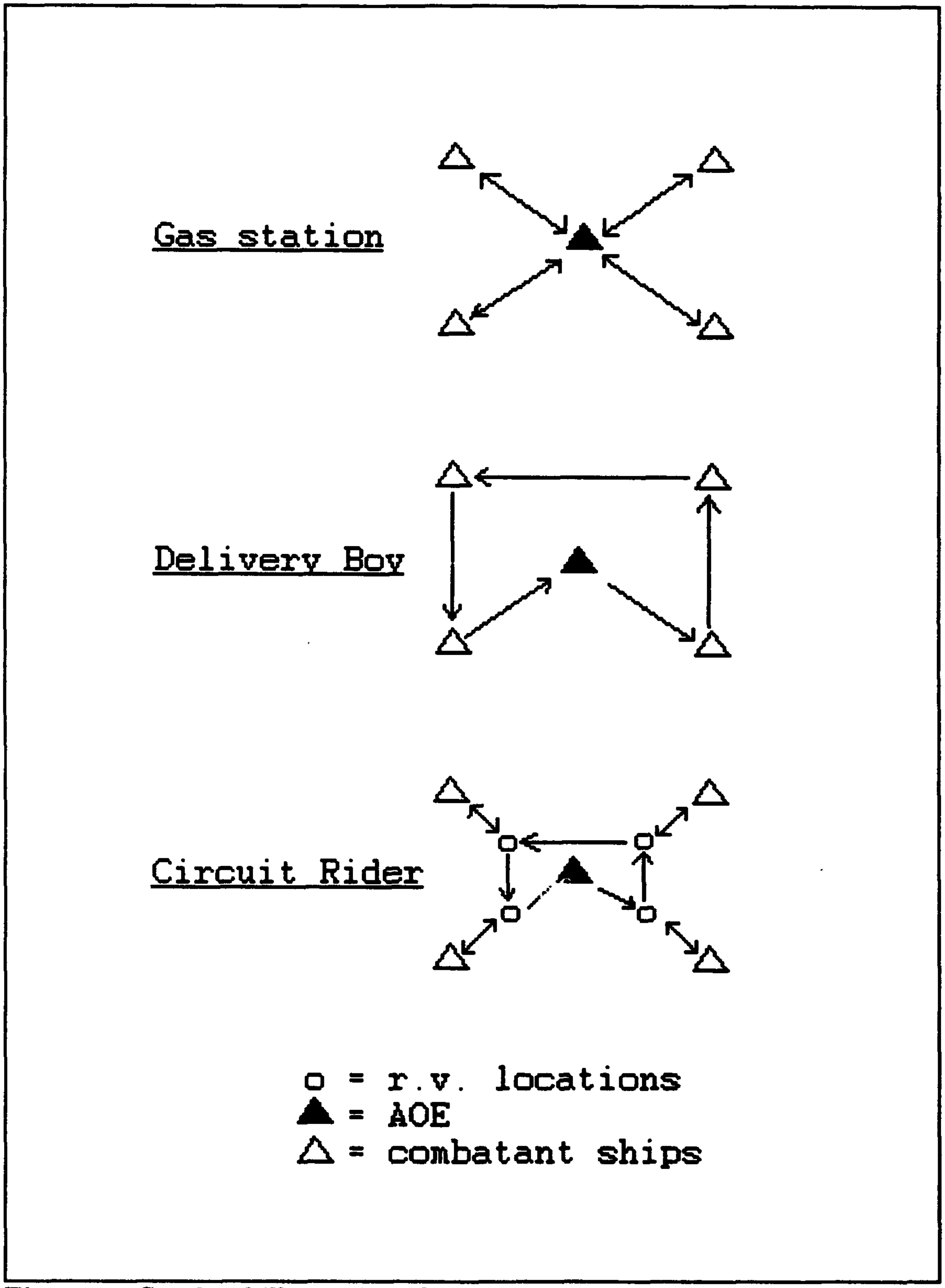


Figure 1 Graphical illustration of unrep tactics

supply ship can unrep multiple ships, e.g., one on the starboard side and one on the port side. Utilizing both CONREP and VERTREP, more than two ships can be replenished simultaneously.

OBJECTIVE: One of the most common objectives is to minimize the time to replenish all ships that require replenishment. On the other hand, e.g., when an air raid is anticipated, a more suitable objective might be to maximize readiness. In this thesis, the readiness is quantified in terms of "combat values" gained by replenishing combatant ships.

OFF-STATION RESTRICTION: When a combatant ship leaves its position in the formation, it is considered to be "off-station". In order not to degrade the integrity of the formation, there may be a requirement that no more than one ship can be off-station during a replenishment cycle.

B. TRAVELING SALESMAN PROBLEM

As stated previously, an unrep plan consists of a sequence and locations for combatant ships to be replenished. Consider first the sequencing part of the planning. Basic to many sequencing problems found in the operations research literature is the TSP. In this problem, a salesman has to visit customers in n cities and he wants to find a sequence in which he can start from his home, visit each city once and only once and return home in the shortest time possible. If one associates ships with cities and the supply ship with the salesman, then the sequence in which he visits the cities corresponds

to the sequence in which the supply ship replenishes the combatant ships. In the case of unrep planning, the time the salesman takes to travel between two cities would depend on the tactic used as well as other factors such as the off-station restriction, the rendezvous locations and time to unrep. For further details on TSP see Lawer et al.[1985]. The subsections below describe three extensions of the basic TSP which are applicable to planning unrep.

1. Multiple Traveling Salesman Problem (M-TSP)

As in the basic TSP, there are n cities to be visited. However, the M-TSP has m salesmen who can visit these cities. The restrictions are that each city must be visited by one and only one salesman and each salesman begins and ends his/her tour from the same (home) city. If all m salesmen have a common home city, then M-TSP can be formulated as a TSP (see, Bellmore and Hong[1974]).

2. Generalized Traveling Salesman Problem (GTSP)

In the GTSP, n cities are separated into k mutually exclusive groups and the salesman has to visit only one city in each of the k groups before he can return home. If $k = n$, then the GTSP reduces to a TSP. General applications of GTSP can be found in, e.g., Henry-Labordere[1969], Sakaena[1970], Laporte et al.[1987], Noon[1988] and Rousseau[1988].

Early approaches for solving GTSP are given in Henry-Labordere[1969], Srivastava et al.[1969] and Sakaena[1970]. More recently,

Laporte and Nobert[1983] and Noon and Bean[1991] proposed branch and bound approach for solving GTSP where the distance between cities are symmetric and asymmetric, respectively.

3. Prize Collecting Traveling Salesman Problem (PCTSP)

Some authors (see, e.g., Golden et al.[1987] and Tsiligirides[1984]) also refer to this problem as the orienteering problem. As in the TSP, assume that there are n cities, however, not all cities need to be visited by the salesman. With each city, there is an associated prize or value which the salesman can 'collect' if he visits the city. The salesman's objective is to visit the subset of cities which provides the maximum prize collection and return home within a given time.

Applications of PCTSP in vehicle routing and inventory problems can be found in Golden et al.[1981, 1984 and 1987]. Balas and Martin[1985], Golden et al.[1987, 1988] and Tsiligirides[1984] describe heuristic procedures for PCTSP. Ramesh et al.[1989] provided an optimal algorithm based on Lagrangean relaxation and problem reformulation. However, PCTSP can also be viewed as a special case of routing and scheduling problems with time windows. These windows indicate the time interval during which the cities can be visited. For PCTSP, the time windows for n cities as well as the salesman's home city are the same. Solomon and Desrosiers[1988] provide a state-of-the-art survey for routing and scheduling with time window. The book by Golden and Assad[1988] also provides many articles on this subject.

C. PRIOR STUDIES ON UNDERWAY REPLENISHMENT

The majority of studies on unrep have focused on simulation models. Only a few provided any guidance or methodology to develop efficient unrep plans. In the United States, there are three existing simulation models: the Replenishment-at-sea Model (RASM), the Battle Force Operation Replenishment Model (BFORM) and the Resupply Sealift Requirements Generator/Ship On-line Scheduler (RSRG/SOS). RASM was developed at the Center for Naval Analyses in 1986. It is written in SIMSCRIPT and considers only the delivery boy and gas station tactics. For a more detailed description of RASM, see Branting[1986]. BEFORM was developed at the Applied Physics Laboratory of Johns Hopkins University and implemented in PASCAL. The model allows three tactics: delivery boy, gas station and a third tactic which generalizes gas station. The details of the model are described in Hereford and Spiegel[1988]. Finally, RSRG/SOS was developed by the David Taylor Research Center in 1988 (see, Vondersmith and Miller[1988] and Melton[1988]). This model is concerned with shuttle lift to a battle group. Williams et al.[1989] and Holder and Gittins[1989] built a simulation model to study the unrep requirements for the British Royal Navy. In all of the above, none discussed the development of an efficient plan. Schrady[1991] wrote that these references either let the safety of replenishment equipment override the unrep planning, myopically sequence ships to be replenished, implement plans created by "intelligent users," or only provide rules of thumb for unrep.

Among those that considered methodologies for scheduling unrep, Hardgrave[1989] showed that, under suitable assumptions, the scheduling of unrep under three tactics (delivery boy, chain saw and circuit rider) can all be formulated as a TSP. Pilnick[1989] and Pilnick et al.[1991] considered replenishment problems using VERTREP. Braunschweig[1991] developed a optimization program to analyze battle group vulnerability using gas station and delivery boy tactics. The combatant off-station time and the length of the minimum replenishment cycle were used as measures of effectiveness for vulnerability. More recently, Zabarouskas[1992] developed two branch and bound algorithms for the delivery boy and circuit rider tactics and Williams[1992] described a heuristic algorithm to schedule unrep for the British Royal Navy.

Other related studies have been completed at the Naval Postgraduate School. Barnaby[1988] used BEFORM to analytically evaluate the trade-offs between delivery boy and gas station tactics. Conley[1988] developed a statistical model to predict replenishment rates and compar[illegible] hem against published rates. Ratliff[1990] modified RAS[illegible] [illegible] e VERTREP. Harris[1989] compared the thre[illegible] [illegible] in United States. Schrady and Wadswort[illegible] [illegible] logistic support system which tracks and predicts the l[illegible] [illegible] us of a battle group and its component units.

III. BASIC MODELS FOR UNREP OPERATION

In this chapter, mixed integer programs are used to model unrep operations under three tactics: delivery boy, circuit rider and gas station. For each tactic, two specific scenarios are considered. The first, the *routine* scenario, is most prevalent while the battle group is in transit or in a relatively peaceful environment. During such situations, the objective is to accomplish replenishment in the most efficient manner. In the models below, efficient refers to the time needed to replenish the battle group. The other scenario, the *rearming* scenario, is motivated by a battle group which expects reattack in, say, a few hours. Most likely, it would be impossible to replenish every ship in the battle group. In this scenario, the objective is to selectively replenish combatant ships which provide the greatest level of strategic readiness before the predicted attack.

To frame the problems, the next section describes the assumptions underlying all models in this chapter. Following these assumptions, models and their relationship to TSPs are described. It is interesting to point out that, unlike many applications of the TSP found in the operations research literature, unrep operations yield TSPs with a relatively small number of cities (or group of cities in the GTSP case). Moreover, unrep operations represent a rich class of TSPs. By varying parameters or features of an unrep operation,

all known extensions of the TSP can be generated. For example, one combination of tactics and scenarios, i.e, circuit rider and the rearming scenario, produces a new extension of the TSP - the generalized orienteering or prize collecting salesman problem.

A. MODEL ASSUMPTIONS

To capture the basic structure and to concisely model unrep problems, the assumptions described below are assumed throughout this chapter. Although some assumptions may be relaxed, their relaxations would obscure the underlying structure and extend the treatment beyond the intended scope of this thesis. Some of these extensions have already been investigated (see, Braunschweig[1991]).

Assumption 1: The supply ship has sufficient supply to replenish all ships in the battle group.

Assumption 2: Both the supply and receiving ships maintain the same formation during unrep. Thus, neither ship falls behind the formation as course and speed remain constant during unrep. Braunschweig[1991] considered fall back in his thesis.

Assumption 3: The time to replenish a given combatant ship is constant and is not affected by the sequence in which unrep occurs. In addition, replenishing a ship below the required level is prohibited.

Assumption 4: Combatant ships are always available to unrep at the designated time. Implicitly, this assumption requires that the decision to begin unrep has been planned with cooperating sea and weather condition in mind. However, by modifying the time window constraints, the supply ship can be forced to unrep combatant ship outside a certain interval, e.g., during an aircraft landing window for an aircraft carrier.

Assumption 5: In both scenarios, it is assumed that an unrep process begins when the first ship to be replenished or the supply ship moves away from its station and ends when all ships including the supply ship have returned to their stations. We refer to the elapsed time between the start and the end of the unrep process as " the total unrep time". Certainly, different methods for measuring total unrep time exist and they can be easily incorporated into the models in this chapter.

B. DELIVERY BOY MODELS

Recall that in delivery boy, the supply ship leaves its station and travels to combatant ships at their stations. In this manner, combatant ships can remain at their stations, thereby maintaining the integrity of the formation near the desire level. Below the delivery boy tactic under the routine scenario is formulated.

INDICES:

b	the initial (or beginning) location of the supply ship;
e	the ending location of the supply ship;
i, j	the position (or location) of combatant ships in the battle group and $i, j = 1, 2, 3, \ldots, n$ (assume that $n \geq 2$);
k	stage number (see explanation below) $k = 1, 2, 3, \ldots, (n+1)$.

DATA:

r_i	replenishing time for ship i;
d_{bi}	travel time from initial position b to ship i;
d_{ij}	travel time from location i to location j;
d_{je}	travel time from ship j to ending position e.

DECISION VARIABLES:

X^{1}_{bi}	equals 1 if the supply ship visits ship i first and equal 0 otherwise;
X^{k}_{ij}	equals 1 if ship i is visited (or replenished) before ship j in stage k and equal 0 otherwise;
$X^{(n+1)}_{je}$	equals 1 if the supply ship visits ship j last and equal 0 otherwise.

<u>Minimum Unrep Time Problem</u>
(Delivery Boy Tactic)

$$Minimize \sum_{i=1}^{n} (d_{bi}+r_i) X^{1}_{bi} + \sum_{k=2}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} (d_{ij}+r_j) X^{k}_{ij} + \sum_{j=1}^{n} d_{je} X^{(n+1)}_{je}$$

subject to

$$\sum_{j=1}^{n} X^{1}_{bj} = 1 \qquad \textbf{(DB-1)}$$

$$X^{1}_{bi} = \sum_{j=1}^{n} X^{2}_{ij} \ , \quad \forall \ i \qquad \textbf{(DB-2)}$$

$$\sum_{j=1}^{n} X^{(k-1)}_{ji} = \sum_{j=1}^{n} X^{k}_{ij} \ , \quad \forall \ k \geq 2, \ i \qquad \textbf{(DB-3)}$$

$$\sum_{j=1}^{n} X^{n}_{ji} = X^{(n+1)}_{ie} \ , \quad \forall \ i \qquad \textbf{(DB-4)}$$

$$\sum_{i=1}^{n} X^{(n+1)}_{ie} = 1 \qquad \textbf{(DB-5)}$$

$$X^1{}_{bi} + \sum_{k=2}^{n} \sum_{j=1}^{n} X^k{}_{ji} = 1 \quad , \quad \forall \quad i \qquad \textbf{(DB-6)}$$

In the above formulation, the sequence of n ships to be visited is replenished in $(n+1)$ stages. During stage 1, the supply ship travels to the first ship in the sequence, during stage $(n+1)$ the supply ship travels from the last ship in the sequence to its ending position, and during the intermediate stages the supply ship travels from one combatant ship to another. Figure 2 graphically displays this stage representation for a formation with three combatant ships. The dotted arrows or arcs indicate one possible unrep sequence. Note that not all sequences of connected arcs from b to e corresponds to a legitimate unrep sequence. For example, a path: b - 2 - 1 - 2 - e is illegal for it visits ship 2 twice.

This representation of the unrep sequences naturally creates more decision variables. A more compact representation without the stage index is shown in Figure 3. However, the first representation is utilized throughout the remainder of this thesis due to its faster computation time when implemented in commercial software packages such as XA (Sunset Software Technology [1985]) and ZOOM (Singhal et al.[1989]).

In the above formulation, b and e may represent the same position, e.g., the supply ship's on-station position inside the formulation, or they may represent two different positions, beginning and ending, as the definition suggests. Moreover, in the definition of the decision variables, there are three

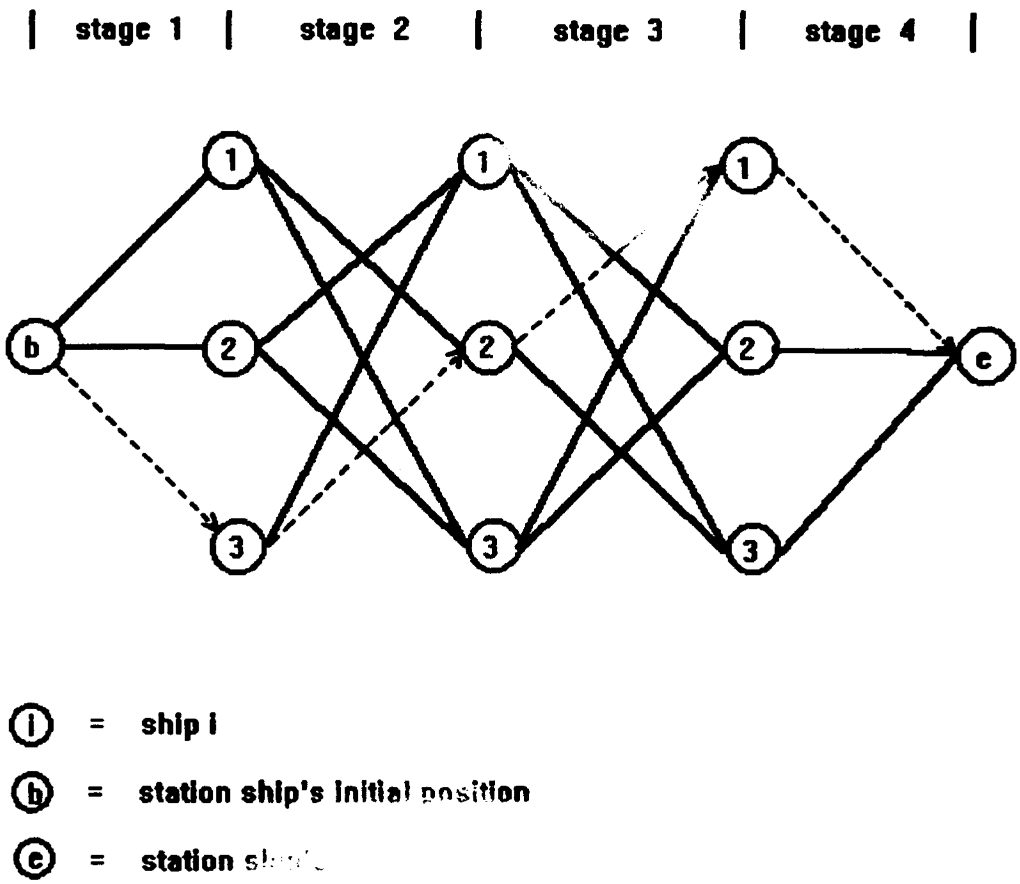


Figure 2 A network structure for the minimum unrep time problem with delivery boy tactic

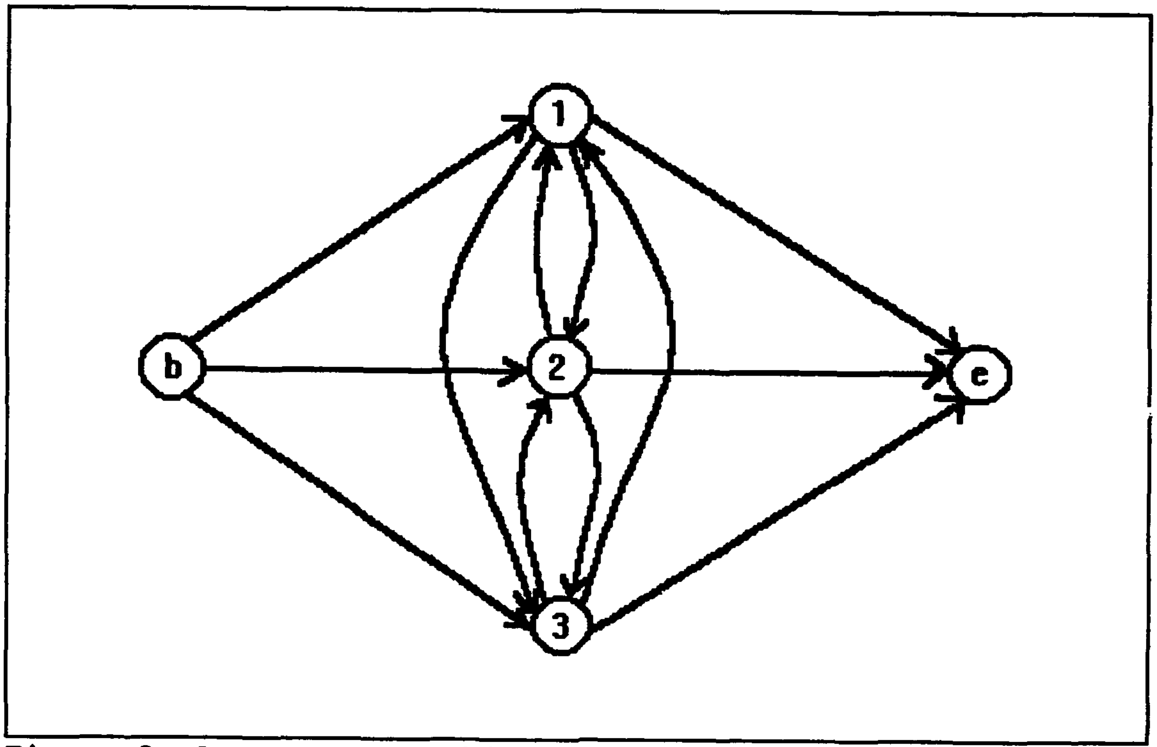


Figure 3 A compact graphical structure of unrep sequence

sets of variables. The first set represents the feasible set of arcs in the first stage, i.e., arc from b to combatant ship i, the second are arcs between combatant ships i and j, and the last from combatant ship i back to e.

The objective consists of three items. The first is the time for the supply ship to travel to the first ship in the sequence plus its unrep time. The second is the travel time and the unrep time for ships in the rest of the sequence. The last term is the time for the supply ship to return to its ending position.

Constraint (DB-1) requires that the supply ship visits only one ship in the first stage. Constraints (DB-2) to (DB-4) simply state that after visiting ship i in stage $(k$-1) the supply ship must proceed to the next ship j in stage k. Observe that (DB-2) and (DB-4) can be viewed as a special case of (DB-3).

Constraints (DB-5) insures that the supply ship returns to its ending position. In the network flow terminology, (DB-1) to (DB-5) simply specify that there is one unit of flow that goes from b to e through the network similar to the one depict in Figure 2. The last constraint, (DB-6), guarantees that each combatant ship is visited once and only once and is sometime referred to as the subtour elimination constraint in the TSP literature. In fact, constraints (DB-1) to (DB-6) precisely describe a traveling salesman tour and the minimum unrep time problem using delivery boy is exactly the basic TSP.

For the rearming scenario, assume that the time available for unrep is h hours. During these h hours, some ships may not be replenished at all. This possibility changes the underlying network structure of the problem. In particular, there are additional arcs connecting node j to node e in every stage signifying that unrep operations may terminate during any stage (see Figure 4). Recall that under this scenario, the objective is to maximize strategic readiness which is taken to be the sum of combat values associated with the replenished ships. Below the formulation for this scenario along with additional data and decision variables are provided. Any notation previously introduced maintains its definition.

ADDITIONAL DATA:

w_i combat value of ship i;

h time available for unrep operations.

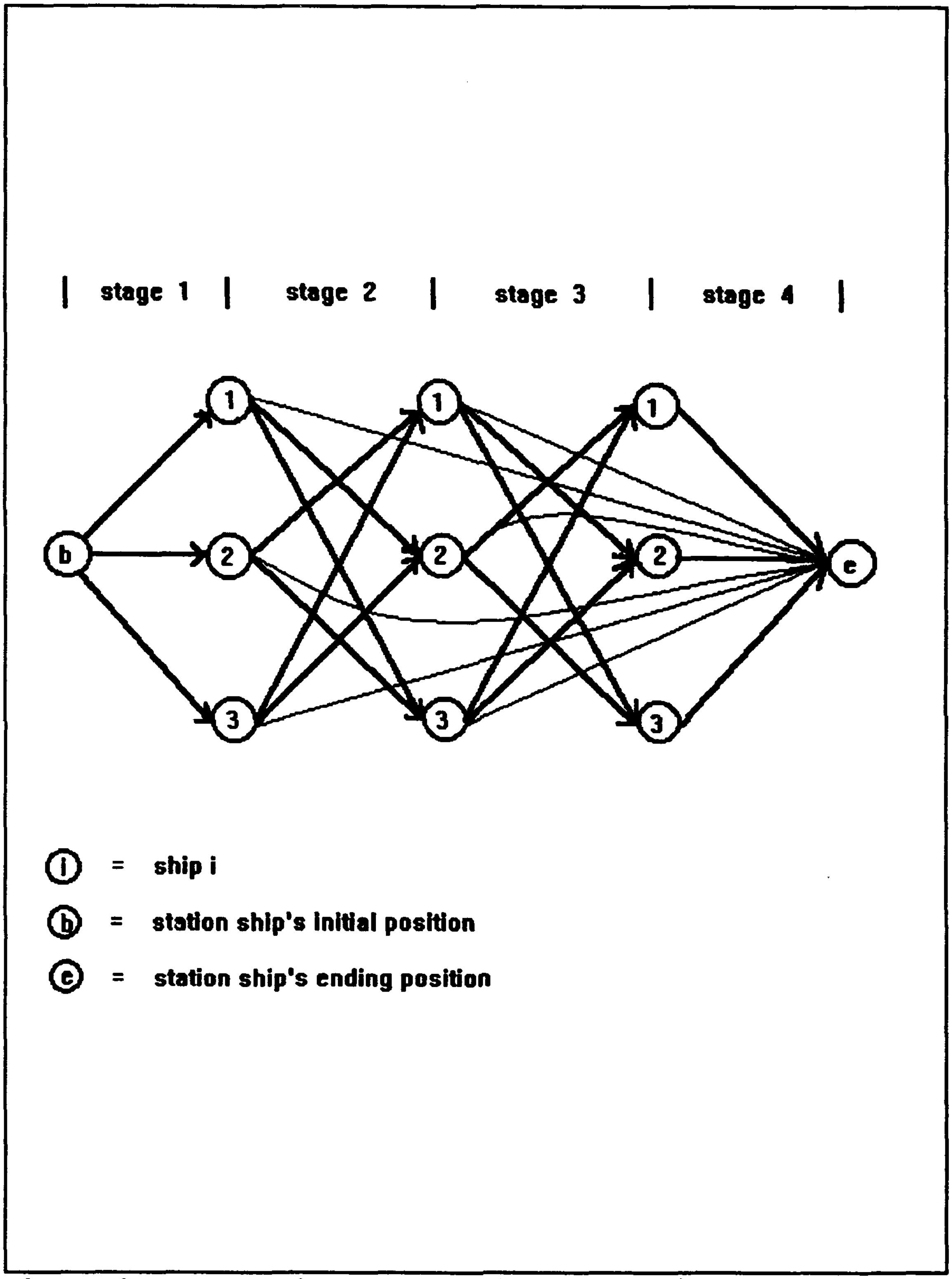


Figure 4 A network structure for the maximum combat value problem with delivery boy tactic

ADDITIONAL DECISION VARIABLES:

V_i equals 1 if ship i is replenished and equals 0 otherwise;

X^{k}_{ie} equals 1 if ship i is replenished last and equals 0 otherwise.

<u>Maximum Combat Value Problem</u>
(Delivery Boy Tactic)

$$\text{Maximize} \quad \sum_{i=1}^{n} w_i V_i$$

subject to

$$\sum_{i=1}^{n} X^{1}_{bi} = 1 \tag{DB-7}$$

$$X^{1}_{bi} = \sum_{j=1}^{n} X^{2}_{ij} + X^{2}_{ie} \quad , \forall \quad i \tag{DB-8}$$

$$\sum_{j=1}^{n} X^{(k-1)}_{ji} = \sum_{j=1}^{n} X^{k}_{ij} + X^{k}_{ie} \, , \quad \forall \; k \geq 3, \; i \tag{DB-9}$$

$$\sum_{j=1}^{n} X^{n}_{ji} = X^{(n+1)}_{ie} \quad , \quad \forall \quad i \tag{DB-10}$$

$$\sum_{k=2}^{(n+1)} \sum_{i=1}^{n} X^{k}_{ie} = 1 \tag{DB-11}$$

$$X^{1}_{bi} + \sum_{k=2}^{n} \sum_{j=1}^{n} X^{k}_{ji} = V_i \quad , \quad \forall \quad i \tag{DB-12}$$

$$\sum_{i=1}^{n} r_i V_i + \sum_{i=1}^{n} d_{bi} X^{1}_{bi} + \sum_{k=2}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} X^{k}_{ij} + \sum_{k=2}^{n+1} \sum_{i=1}^{n} d_{ie} X^{k}_{ie} \leq h \tag{DB-13}$$

The objective function seeks to maximize the total combat value of replenished ships. The constraint sets (DB-7) to (DB-11) are analogous to (DB-1) to (DB-5). Any additional term involving X^{k}_{ie} allows the supply ship to return to its ending position at any stage. Constraints (DB-12) is analogous to (DB-6) except the right hand side of (DB-12) has V_i instead of 1. In this manner, V_i signifies that a flow of one unit enters node i only if unrep is performed. The last constraint ensures that the unrep operations does not run beyond the available time, h. The first term in (DB-13) is the total unrep time and the last three terms are the travel time for the supply ship.

Observe that the maximum combat value problem using the delivery boy tactic fits the description of the orienteering or prize collecting TSP. As in the PCTSP, the maximum combat value problem allows the supply ship (i.e., the traveling salesman) to visit a subset of cities (ships) in order to maximize the combat value (prizes) and return home by time h.

C. CIRCUIT RIDER MODELS

The delivery boy tactic requires each combatant ship to be replenished at its on-station location which is assumed to consist of only one point. In circuit rider, this assumption is relaxed to allow for more flexibility. In particular, each combatant ship can unrep at more than one locations which are referred to as 'rendezvous (r.v.) locations'. For simplicity, it is assumed that every ship has exactly m r.v. locations. Figure 5 displays the network structure for an

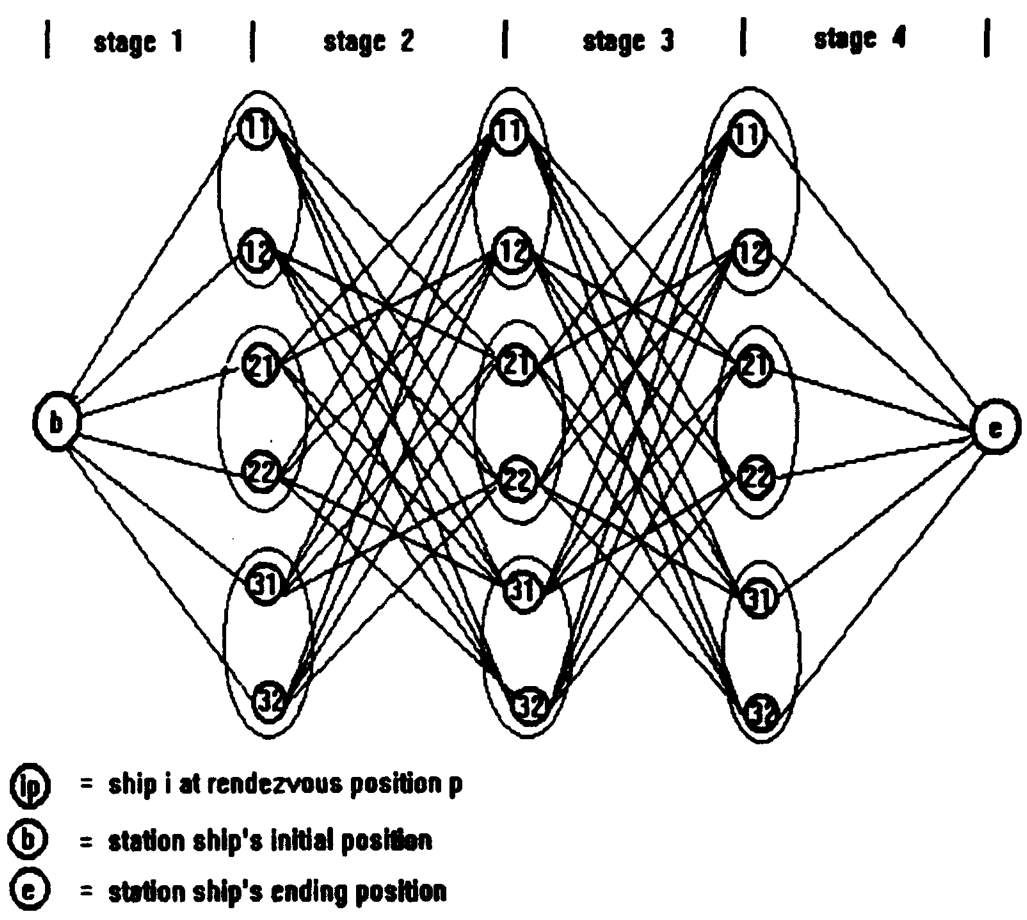


Figure 5 A network structure for the minimum unrep time problem with circuit rider tactic

unrep problem with three ships where each ship has exactly two r.v. locations. Note that two ships can be replenished simultaneously if the r.v. locations for two different ships coincide. In the routine scenario, the problem involves selecting the r.v. location for each ship as well as selecting the sequence in which to unrep the ship. To formulate the problem, define the following additional indices and data.

ADDITIONAL INDICES:

p,q denote a rendezvous location; $p,q = 1,2,...,m$.

ADDITIONAL DATA:

$d_{b(i,p)}$ travel time from the beginning position to ship i at r.v. location p;

$d_{(i,p)(j,q)}$ travel time from ship i at r.v. location p to ship j at r.v. location q;

$d_{(i,p)e}$ travel time from ship i at r.v. location p to the ending position.

NEW DECISION VARIABLES:

$X^{1}_{b(i,p)}$ equals 1 if ship i is replenished first at r.v. location p and equals 0 otherwise;

$X^{k}_{(i,p)(j,q)}$ equals 1 if ship i is replenished at r.v. location p directly before ship j is replenished at r.v. location q during stage k and equals 0 otherwise;

$X^{(n+1)}_{(i,p)e}$ equals 1 if ship i is replenished last at r.v. location q and equals 0 otherwise;

$V_{(i,p)}$ equals 1 if ship i is replenished at r.v. location p and equals 0 otherwise.

The problem under the routine scenario can be stated as follows.

<u>Minimum Unrep Time Problem</u>
(Circuit Rider Tactic)

$$Minimize \sum_{i=1}^{n}\sum_{p=1}^{m}(d_{b(i,p)}+r_i)X^1{}_{b(i,p)}$$
$$+\sum_{k=2}^{n}\sum_{i=1}^{n}\sum_{p=1}^{m}\sum_{j=1}^{n}\sum_{q=1}^{m}(d_{(i,p)(j,q)}+r_j)X^k{}_{(i,p)(j,q)}+\sum_{j=1}^{n}\sum_{q=1}^{m}d_{(j,q)\bullet}X^{(n+1)}{}_{(j,q)\bullet}$$

subject to

$$\sum_{j=1}^{n}\sum_{q=1}^{m}X^1{}_{b(j,q)}=1 \qquad \text{(CR-1)}$$

$$X^1{}_{b(i,p)}=\sum_{j=1}^{n}\sum_{q=1}^{m}X^2{}_{(i,p)(j,q)}\,,\quad \forall\ i,\ p \qquad \text{(CR-2)}$$

$$\sum_{j=1}^{n}\sum_{q=1}^{m}X^{(k-1)}{}_{(j,q)(i,p)}=\sum_{j=1}^{n}\sum_{q=1}^{m}X^k{}_{(i,p)(j,q)}\,,\ \forall\ k\geq 3,\ i,\ p \qquad \text{(CR-3)}$$

$$\sum_{j=1}^{n}\sum_{q=1}^{m}X^n{}_{(j,q)(i,p)}=X^{(n+1)}{}_{(i,p)\bullet}\,,\ \forall\ i,\ p \qquad \text{(CR-4)}$$

$$\sum_{i=1}^{n}\sum_{p=1}^{m}X^{(n+1)}{}_{(i,p)\bullet}=1 \qquad \text{(CR-5)}$$

$$X^1{}_{b(i,p)}+\sum_{k=2}^{n}\sum_{j=1}^{n}\sum_{q=1}^{m}X^k{}_{(j,q)(i,p)}=V_{(i,p)}\,,\ \forall\ i,\ p \qquad \text{(CR-6)}$$

$$\sum_{p=1}^{m}V_{(i,p)}=1\,,\quad \forall\ i \qquad \text{(CR-7)}$$

The objective function and constraints (CR-1) to (CR-5) are analogous to those of the delivery boy tactic. Constraint (CR-6) differs from (DB-6) because variable $V_{(i,p)}$ is needed to indicate the r.v. location of ship i. Constraint (CR-7) ensures that each ship uses only one r.v. location.

Observe that the above formulation corresponds to that of a GTSP. In the circuit rider, each r.v. location can be considered as a city in the GTSP. These cities (r.v. locations) are then grouped into n groups, one for each ship. Then, the salesman must visit one and only one city in each of the n groups and return home, which matches the minimum unrep time problem with the circuit rider tactic.

For the rearming scenario, the problem of maximizing combat value with a circuit rider tactic becomes a generalization of the orienteering or the prize collecting TSP. The prize collecting TSP requires the salesman to visit a subset of n cities in order to maximize the value of the collected prizes while returning to his/her home city in a given amount of time, h. The model below for circuit rider generalizes the "n cities" to "n group of cities". In the framework of underway replenishment, a group of cities represent a group of r.v. locations for a combatant ship. Moreover, the salesman collects only one prize by visiting a city (or r.v. location) in a given group (or of a given ship). So, visiting an additional city in the same group yields no extra prize. This restriction translates to not allowing the same ship to be replenished twice at two different r.v. locations. This generalization has not been previously

addressed in the literature and is modeled below in a manner similar to the same problem with the delivery boy tactic.

Maximum Combat Value Problem
(Circuit Rider Tactic)

Maximize $$\sum_{i=1}^{n}\sum_{p=1}^{m} w_i V_{(i,p)}$$

subject to

$$\sum_{j=1}^{n}\sum_{q=1}^{m} X^1{}_{b(j,q)} = 1 \qquad \text{(CR-8)}$$

$$X^1{}_{b(i,p)} = \sum_{j=1}^{n}\sum_{q=1}^{m} X^2{}_{(i,p)(j,q)} + X^2{}_{(i,p)e}\,, \quad \forall\ i,\ p \qquad \text{(CR-9)}$$

$$\sum_{j=1}^{n}\sum_{q=1}^{m} X^{(k-1)}{}_{(j,q)(i,p)} = \sum_{j=1}^{n}\sum_{q=1}^{m} X^k{}_{(i,p)(j,q)} + X^k{}_{(i,p)e}\,, \quad \forall\ k\geq 3,\ i,\ p \qquad \text{(CR-10)}$$

$$\sum_{j=1}^{n}\sum_{q=1}^{m} X^n{}_{(j,q)(i,p)} = X^{(n+1)}{}_{(i,p)e}\,, \quad \forall\ i,\ p \qquad \text{(CR-11)}$$

$$\sum_{k=2}^{n+1}\sum_{i=1}^{n}\sum_{p=1}^{m} X^k{}_{(i,p)e} = 1 \qquad \text{(CR-12)}$$

$$X^1{}_{b(i,p)} + \sum_{k=2}^{n}\sum_{j=1}^{n}\sum_{q=1}^{m} X^k{}_{(j,q)(i,p)} \ \text{[illegible]})\,, \quad \forall\ i,\ p \qquad \text{(CR-13)}$$

$$\sum_{p=1}^{m} V_{(i,p)} = 1\,, \quad \forall\ i \qquad \text{(CR-14)}$$

$$\sum_{i=1}^{n} \sum_{p=1}^{m} r_i V_{i,p} + \sum_{i=1}^{n} \sum_{p=1}^{m} d_{b(i,p)} X^1{}_{b(i,p)} + \sum_{k=2}^{n} \sum_{i=1}^{n} \sum_{p=1}^{m} \sum_{j=1}^{n} \sum_{q=1}^{m} d_{(i,p)(j,q)} X^k{}_{(i,p)(j,q)} + \sum_{k=2}^{n+1} \sum_{i=1}^{n} \sum_{p=1}^{m} d_{(i,p)e} X^k{}_{(i,p)e} \leq h \tag{CR-15}$$

With the exception of the subscripts, the above objective function and constraints are similar to those for the maximum combat value problem with the delivery boy tactic. As before, (CR-13) and (CR-14) ensure that each ship is replenished at only one r.v. location.

D. GAS STATION MODELS

Unlike the delivery boy and circuit rider tactics, gas station admits several optimization models/problems instead of just one for each of the two scenarios considered thus far. This is due to the flexibility inherent in the tactic. Under the gas station tactic, more than one ship can be replenished simultaneously alongside the supply ship. Typically, a supply ship can replenish two combatant ships, one on the starboard side and the other on the port side. For simplicity sake, this thesis refers to such a supply ship as having two "transfer stations". In addition, because the supply ship is stationary relative to the moving formation, the combatant ships must leave their positions to receive supplies alongside the supply ship. This degrades the integrity of the formation. In the extreme case, all ships leave their positions to form a "gasoline alley" (see, Braunschweig [1991]). In this section, the following cases are considered.

1) A limited number of ships off-station: If the supply ship has m transfer stations, then at most m ships are allowed off-station at any time.

2) No off-station restriction: There is no limit on the number of ships that can be off-station. However, it is assumed that the practice of forming a "gasoline alley" is prohibited. Instead, ships are assumed to wait for their turn at their stations. They are allowed to leave when their arrivals at the supply ship coincide with another ship finishing its unrep.

1. Limited number of ships off-station

Under the routine scenario, having m transfer stations can be viewed as having m supply ships, each with one transfer station. If the objective is to minimize the sum or average of the total unrep time at the m transfer stations, then resulting optimization problem is a M-TSP. However, it is more appropriate to minimize the maximum total unrep time at the m transfer stations, as this represents the time at which the supply ship completes its task and all ships returned to their stations. To state the problem in the framework of a M-TSP, the following are new definitions of indices and variables.

INDICES:

a transfer station $a = 1,\ldots, m$, where m = number of transfer stations;

k stage $k = 1,\ldots, K$ (see discussion below).

DECISION VARIABLES:

F time when all ships are back to their on-station locations;

$X^{(a,k)}_{ij}$ equals 1 if ship i is replenished before ship j during stage k at transfer station a and equals 0 otherwise;

$X^{(a,1)}_{bj}$ equals 1 if ship j is the first ship to replenished at transfer station a and equals 0 otherwise;

$X^{(a,k)}_{je}$ equals 1 if ship j is the last ship to be replenished at transfer station a and equals 0 otherwise.

Below is one formulation of the gas station tactic with m transfer stations (A different formulation is provided in Appendix A).

Minimum Unrep Time Problem

(Gas station with limited number of ships off-station)

$$\text{Minimize} \quad F$$

subject to

$$\sum_{j=1}^{n} X^{(a,1)}_{bj} = 1 \quad , \quad \forall a \tag{GS-1}$$

$$X^{(a,1)}_{bi} = \sum_{j=1}^{n} X^{(a,2)}_{ij} + X^{(a,2)}_{ie} \, , \quad \forall \, a, \, i \tag{GS-2}$$

$$\sum_{j=1}^{n} X^{(a,k-1)}_{ji} = \sum_{j=1}^{n} X^{(a,k)}_{ij} + X^{(a,k)}_{ie} \, , \; \forall \, k \geq 3, \, a, \, i \tag{GS-3}$$

$$\sum_{j=1}^{n} X^{(a,k-1)}_{ji} = X^{(a,k)}_{ie} \, , \quad \forall \, a, \, i \tag{GS-4}$$

$$\sum_{k=2}^{K} \sum_{i=1}^{n} X^{(a,k)}_{ie} = 1 \quad , \quad \forall \; a \tag{GS-5}$$

$$\sum_{a=1}^{m} X^{(a,1)}_{bi} + \sum_{a=1}^{m} \sum_{k=2}^{K} \sum_{j=1}^{n} X^{(a,k)}_{ji} = V_i \, , \quad \forall \, i \tag{GS-6}$$

$$\sum_{i=1}^{n}(d_{ib}+r_i)X^{(a,1)}{}_{bi}+\sum_{k=2}^{K}\sum_{i=1}^{n}\sum_{j=1}^{n}(d_{bi}+d_{jb}+r_j)X^{(a,k)}{}_{ij} + \sum_{k=2}^{K}\sum_{j=1}^{n}d_{bj}X^{(a,k)}{}_{je} \le F\,,\qquad \forall\, a \tag{GS-7}$$

In the above definition, the index k ranges from 1 to K. If $m = 1$, the value of K must be set to $(n+1)$. When $m \geq 2$, each transfer station must replenish at least one ship and can replenish at most $(n\text{-}m+1)$ ships. To ensure an optimal solution, K should be set to $(n\text{-}m+1)$. However, this value of K generates the maximum number of variables in the above formulation. If the combatant ships require similar amount of supplies and approximately the same amount of time to travel to and from the supply ship, then a good choice of K is the ceiling of (n/m).

For the above problem, the constraints (GS-1) to (GS-6) are conceptually the same as (DB-7) to (DB-12). The left hand side of (GS-7) computes the time each transfer station finishes replenishing ships and the variable F simply represents the maximum of these finishing times among the m transfer stations.

When there is only one transfer station, and only one ship is allowed off-station, Hardgrave[1989] showed that all sequences yield the same total unrep time. Thus, the above problem is unnecessary when $m = 1$.

Consider now the maximum combat value problem with limitations on the number of ships allowed off-station. The model below uses the following definition of decision variables and is stated for $a = 1, 2$.

ADDITIONAL DECISION VARIABLES:

V^a_i equals 1 if ship i is replenished at transfer station a.

<u>Maximum Combat Value Problem</u>
(Gas Station with unlimited number of ships off-station)

$$Maximize \quad \sum_{a=1}^{m}\sum_{i=1}^{n} w_i V^a{}_i$$

subject to constraints (GS-1) - (GS-5) and

$$X^{(a,1)}{}_{b} + \sum_{k=2}^{K}\sum_{j=1}^{n} X^{(a,k)}{}_{ji} = V^a{}_i \ , \quad \forall\ a,\ i \qquad \text{(GS-8)}$$

$$\sum_{a=1}^{m} V^a{}_i \le 1 \quad , \qquad \forall\ \ i \qquad \text{(GS-9)}$$

$$\sum_{i=1}^{n} (d_{ib}+r_i) X^{(a,1)}{}_{bi} + \sum_{k=2}^{K}\sum_{i=1}^{n}\sum_{j=1}^{n} (d_{bi}+d_{jb}+r_j) X^{(a,k)}{}_{ij} + \sum_{k=2}^{K}\sum_{j=1}^{n} d_{bj} X^{(a,k)}{}_{je} \le F \ , \qquad \forall\ \ a \qquad \text{(GS-10)}$$

The combination of (GS-8) and (GS-9) allow each ship to be replenished at most once at only one transfer station. Constraint (GS-10) ensures that all transfer stations complete their task by time h.

2. Unlimited number of ships off-station

To facilitate the formulation of the optimization problem, it is assumed without loss of generality that the time of the first replenishment at any transfer station is at time zero. Note that this does not mean that every

transfer station starts replenishing a ship at time zero. In fact, to achieve minimum unrep time, it is sometimes necessary to start replenishing after time zero at some transfer stations. Figure 6 illustrates an example in which it is desirable to have a transfer station begin replenishing ships after time zero. In Figure 6, there are two ships, A and B, to be replenished and the supply ship has two transfer stations. Making ships A and B replenish at time zero yields a total unrep time of 7. However, when ship B is allowed to replenish later, the total unrep time reduces to 6.

In the formulation below, the time each transfer station begins replenishing ships is denoted as $g^{(a,1)}$ where a indicates the transfer station. In Figure 8, $g^{(1,1)} = 0$ and $g^{(2,1)} = 1$. In words, $g^{(a,1)}$ is the difference (or gap) between the start of the replenishment process at station a and the time of the first replenishment at any station (which is assumed to be zero).

The concept of assuming that the first replenishment begins at time zero is extended to replenishing the next ship in the sequence. In particular, it would be ideal to have the transfer station replenish ships one after the other, i.e., without any gap. However, this is not always optimal. Figure 7 illustrates a counter example with 2 combatant ships and one transfer station.

Below is the statement of the problem where *TF-TS* represents the time needed for the unrep operation. As before, notation not explicitly defined maintains its previous definition.

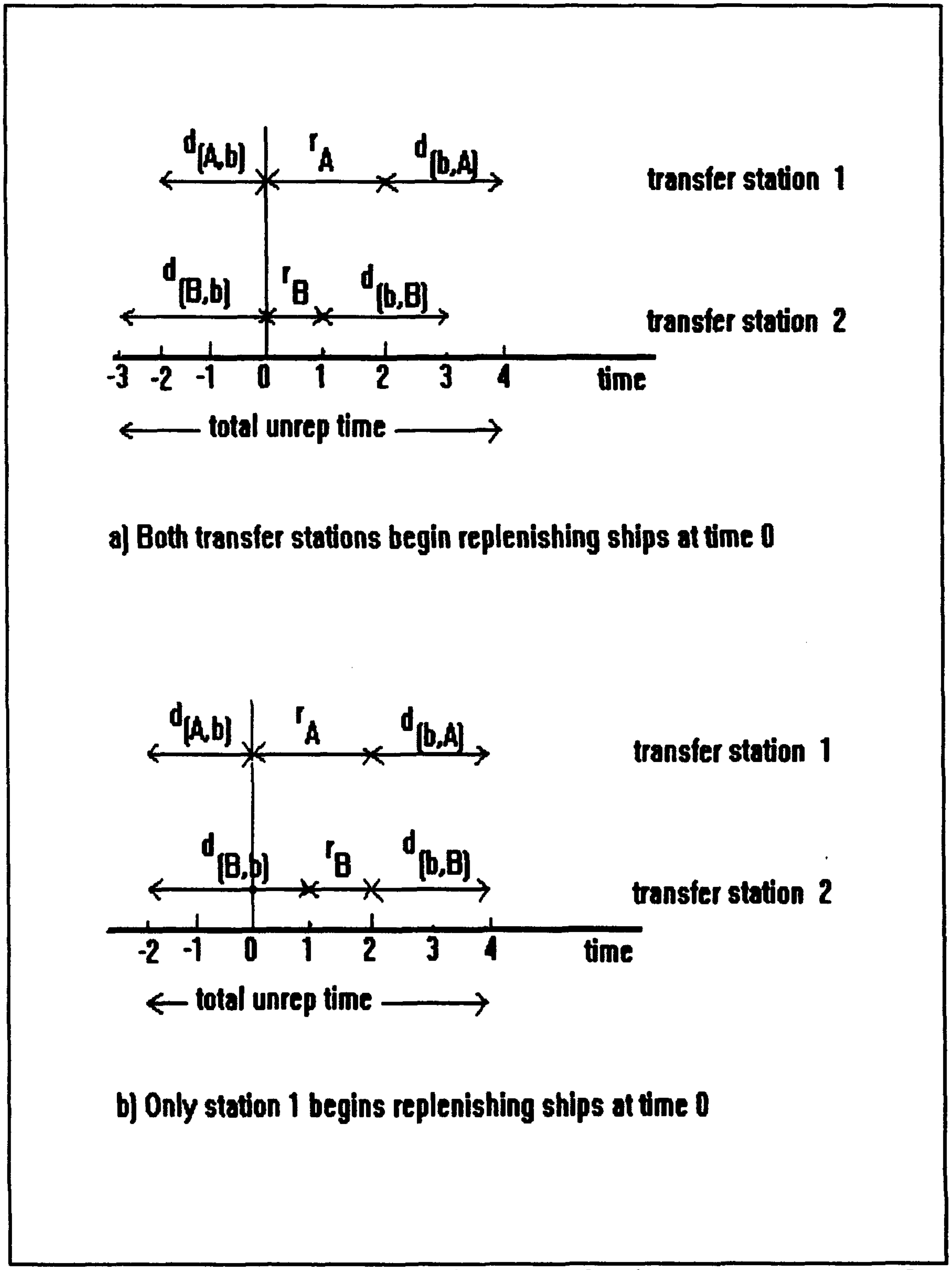


Figure 6 : Gaps for the first replenishment at transfer stations

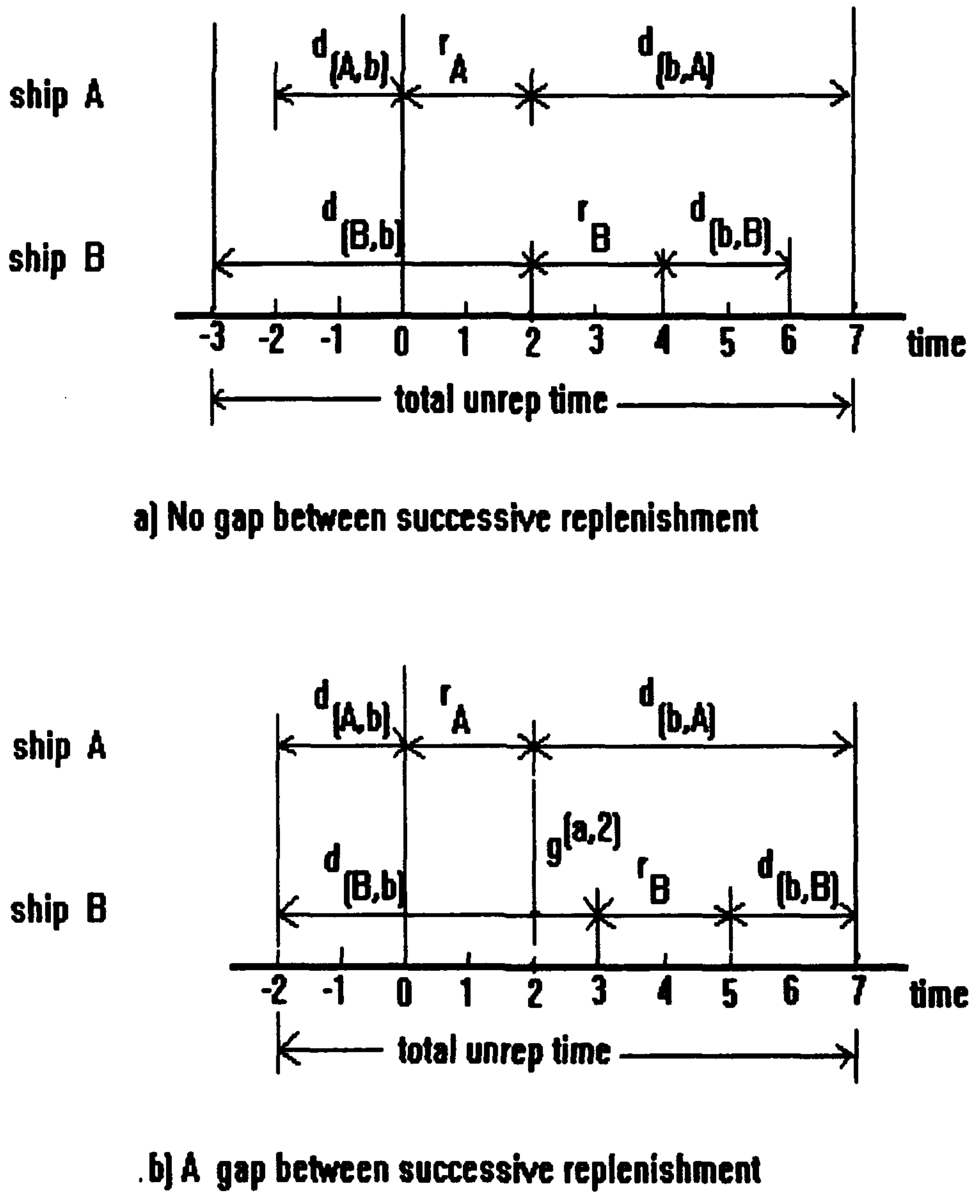


Figure 7 : Gaps between successive replenishment

ADDITIONAL DECISION VARIABLES:

TS the start time of the unrep process;

TF the finish time of the unrep process;

$G^{(a,k)}$ gap at transfer station a during stage k, where $k = 1,...,K$.

<u>Minimum Total Unrep Time</u>

(Gas station with unlimited number of ships off-station)

$$\text{Minimize} \quad TF - TS$$

subject to

$$-\sum_{j=1}^{n} d_{jb}X^{(a,1)}{}_{bj} + g^{(a,1)} \geq TS \ , \quad \forall \ a \qquad \text{(GS-11)}$$

$$\sum_{j=1}^{n} r_j X^{(a,1)}{}_{bj} + g^{(a,1)} - \sum_{i=1}^{n}\sum_{j=1}^{n} d_{jb}X^{(a,2)}{}_{ij} + g^{(a,2)} \geq TS \ , \ \forall \ a \qquad \text{(GS-12)}$$

$$\sum_{j=1}^{n} r_j X^{(a,1)}{}_{bj} + g^{(a,1)} + \sum_{l=2}^{k-1}\{\sum_{i=1}^{n}\sum_{j=1}^{n} r_j X^{(a,l)}{}_{ij} + g^{(a,l)}\}$$
$$- \sum_{i=1}^{n}\sum_{j=1}^{n} d_{jb}X^{(a,k)}{}_{ij} + g^{(a,k)} \geq TS \ , \ \forall \ k\geq 3, \ a$$

$$\text{(GS-13)}$$

$$\sum_{j=1}^{n} (d_{bj}+r_j) X^{(a,1)}{}_{bj} + g^{(a,1)} \leq TF \ , \quad \forall \ a \qquad \text{(GS-14)}$$

$$\sum_{j=1}^{n} r_j X^{(a,1)}{}_{bj}+g^{(a,1)}+\sum_{i=1}^{n}\sum_{j=1}^{n} (d_{bj}+r_j) X^{(a,2)}{}_{ij}+g^{(a,2)} \leq TF, \ \forall \ a \qquad \text{(GS-15)}$$

$$\sum_{j=1}^{n} r_j X^{(a,1)}{}_{bj} + g^{(a,1)} + \sum_{l=2}^{k-1} \{ \sum_{i=1}^{n} \sum_{j=1}^{n} r_j X^{(a,l)}{}_{ij} + g^{(a,l)} \}$$
$$+ \sum_{i=1}^{n} \sum_{j=1}^{n} (d_{bj} + r_j) X^{(a,k)}{}_{ij} + g^{(a,k)} \le TF \quad , \quad \forall\, k \ge 3,\ a \tag{GS-16}$$

$$TS \le 0 \ , \ TF \ge 0 \ , \ g^{(a,k)} \ge 0 \ , \ \forall\, a,\ k \tag{GS-17}$$

The left hand side of constraint (GS-11) computes the time the first ship replenished at station a must start its travel toward the supply ship. For example, if ship j is the first in the sequence at station a and there is no gap, then ship j must begin its travel to the supply ship at time $-d_{jb}$. Similarly, the left hand side of constraints (GS-12) and (GS-13) compute the time that the k^{th} ship to be replenished at station a must start its travel toward the supply ship. To illustrate, assume that ship 1, 2 and 3 are replenished successively at station a (see Figure 8). Then, ship 3 must begin its travel toward the supply ship at time $(r_1 + g^{(a,1)}) + (r_2 + g^{(a,2)}) + g^{(a,3)} - d_{3b} = 0$. Thus, TS represents theearliest start time among n ships. Constraints (GS-14) - (GS-16) are analogous to (GS-11) - (GS-13) and, in combination, they compute TF which is the time needed for all ships to return to their stations.

Using the constraints and variables described thus far, the problem rearming scenario can be stated as following

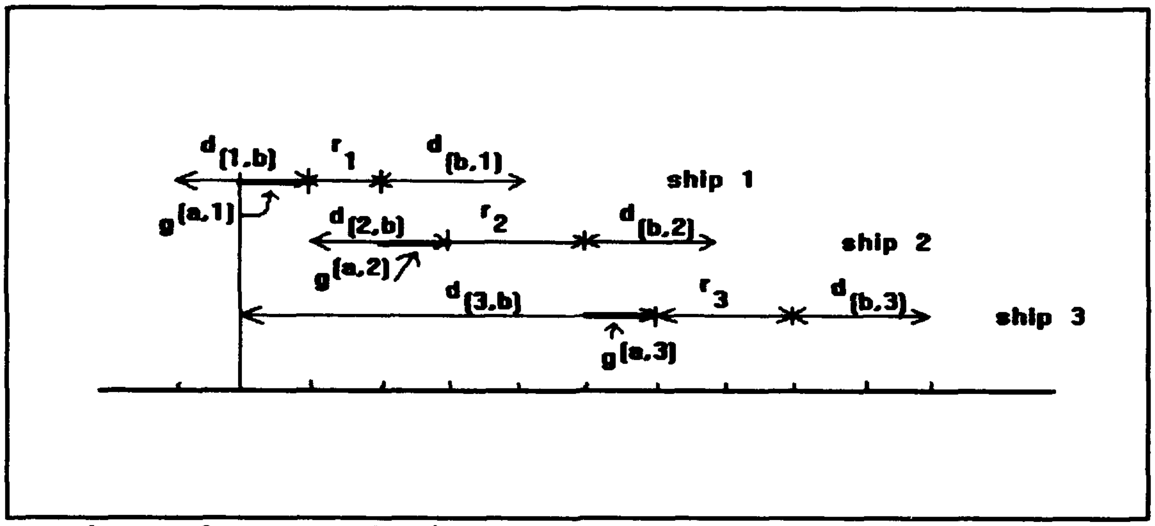


Figure 8 : A replenishment sequence at a transfer station

Maximum Combat Value Problem
(Gas station with unlimited number of ships off-station)

$$\textit{Maximize} \quad \sum_{a=1}^{m}\sum_{i=1}^{n} w_i V^a{}_i$$

subject to (GS-1) - (GS-5), (GS-8), (GS-9), (GS-11) - (GS-17) and the following

$$TF - TS \le h \qquad \textbf{(GS-18)}$$

Recall that *TF* and *TS* are computed in constraints (GS-11) to (GS-17).

To summarize, the models for gas station tactic, particularly those without any off-station restriction, represent generalizations of TSP which are unique to underway replenishment operations. Although these models may involve no more than 15 ships (future battle groups may contain between four and seven combatant ships), some contain a rather large number of (binary) decision variables. As demonstrated in the next chapter, these

models/problems can require a large amount of cpu time to obtain an optimal solution using available commercial software.

IV. COMPUTATIONAL EXPERIENCE

Four battle group formations from the literature (Barnaby[1988] and Hardgrave[1989]) were utilized to validate models in Chapter III and to study the effects of various tactics and scenarios. Data concerning the formations are summarized in Table 2 (see Appendix B for graphical displays of the four formations). In Table 2, there are five columns of data for each formation. The first column provides the names of the ships. The second and third columns provide ship positions in term of the relative bearing (BR) in degrees and range (RG) in nautical miles, respectively. Note that it is assumed that AOE is the supply ship in all four formations. The fourth column (Unrep Time) gives the time to refuel (or replenish) each ship in hours. These times are based on (i) the current amount of fuel on board (which were randomly generated), (ii) the assumption that every ship is to be refueled to 75% of the capacity listed in the *'The Naval Institute Guide to Combat Fleets of the World 1990/1991'* and (iii) the transfer rates from Barnaby[1988]. The final column provides the randomly assigned combat values for each ship. For the circuit rider tactic, combatant ships, i.e., all ships except the aircraft carrier, have three additional on-station positions that are randomly selected within ± 10 degrees and ± 10 nautical miles from the position listed in the second and third

TABLE 2. INPUT FORMATION DATA SETS

Formation #1

Ship	Br	Rg	UNREP Time	Combat Value
AOE	180	03	—	—
CV63	000	00	4.7	10
CG47	040	10	2.6	8
DDG51	320	10	2.3	6
DD963A	030	60	2.6	6
DD963B	330	60	2.6	6
FFG7	180	03	1.4	4

Formation #2

Ship	Br	Rg	UNREP Time	Combat Value
AOE	180	03	—	—
CV63	000	00	4.7	10
CG47	000	10	2.6	8
DDG51A	090	60	2.3	6
DDG52A	270	60	2.3	6
DD963A	030	60	2.6	6
DD963B	330	60	2.6	6
DD963C	180	10	2.6	6
FFG7A	090	10	1.4	4
FFG7B	090	10	1.4	4

Formation #3

Ship	Br	Rg	UNREP Time	Combat Value
AOE	180	03	—	—
CV63	000	00	4.7	10
CG47A	000	10	2.6	8
CG47B	000	60	2.6	8
CG26A	080	60	2.6	8
CG26B	280	60	2.6	8
DDG51A	130	60	2.3	6
DDG51B	230	60	2.3	6
DD963A	040	43	2.6	6
DD963B	320	60	2.6	6
DD963C	180	10	2.6	6
FFG7A	090	10	1.4	4
FFG7B	270	10	1.4	4

Formation #4

Ship	Br	Rg	UNREP Time	Combat Value
AOE	180	03	—	—
CV63	000	00	4.7	10
CG16	000	160	2.6	8
CG47	060	10	2.6	8
DDG37	320	60	2.3	6
DDG2	040	60	2.3	6
DD963A	310	60	2.6	6
DD963B	045	70	2.6	6
FFG7	220	10	1.4	4
FF1052	350	160	1.4	4

columns. To obtain the travel times for either the AOE or the combatant ships, it is assumed that the formation speed is 15 kts and the ship speed for both AOE and combatant ship is 26 kts. The formula to calculate travel time from (Euclidean) coordinate (x_i,y_i) to (x_j,y_j) is as follows (see, Hardgrave[1989]):

$$TravelTime=\frac{F\times(y_j-y_i)+\{[F\times(y_j-y_i)]^2+(S^2-F^2)\times[(x_j-x_i)^2+(y_j-y_i)^2]\}^{1/2}}{(S^2-F^2)}$$

where F (formation speed) = 15 and S (ship speed) = 26.

The two sections below report computational results for the two scenarios: routine and rearming. For each scenario, all three tactics, delivery boy (DB), circuit rider (CR) and gas station, are considered. For gas station, there are four variations. The first and second variations, L1 and U1, have one transfer station; however, L1 allows at most one ship to be off-station and U1 allows an unlimited number of ships to be off-station. Similarly, the next two variations, L2 and U2, have two transfer stations; however, L2 allows at most two ships to be off-station and U2 allows an unlimited number. All models were implemented in GAMS (Brooke et al.[1988]) and the resulting mixed integer programming (MIP) problems were solved by a program developed by Sunset Software Technology called XA (see Appendix C for GAMS listings of all the models). The computing (cpu) times are based on an IBM PS/2 personal computer model P-70 with an Intel-386 (20 MHz) processing chip and an Intel 387 math-coprocessor. Most problems are solved to within ten percent of the optimal solution, i.e., OPTCR is set to 0.1 in GAMS, or until the time limit of

three hours is exceed, i.e., RESLIM = 10800 seconds. Exceptions to these criteria are indicated in the tables summarizing various results. To speed up the solution time of some problems, in particular problems with tactic CR and U2, additional restrictions and constraints (valid inequalities) are utilized to reduce the number of (binary) decision variables and to improve the bounds provided by the linear programming relaxation.

A. ROUTINE SCENARIO

Recall that, during a routine operation, the objective is to minimize the time to replenish all ships in the formation. Table 3 summarizes the computational results. Figure 9 graphically displays the cpu times for the five possible tactics. These figures point out that tactic CR and U2 are much more difficult to solve by standard MIP solvers. Figure 10 compares unrep times across the different tactics. As expected, U2 provides the quickest unrep time over all four formations. Note also that CR is only slightly better than DR. This is due on part by the limitation imposed on the model. First, there are only four possible rendezvous points for each combatant ship, except the aircraft carrier which only has one. Second, all rendezvous positions are relatively near the on-station positions listed in the second and third columns in Table 2. Finally because of the time limit of three hours, some of problems

TABLE 3. MINIMUM UNREP TIME & CPU TIME FOR FORMATIONS

Formation (# ships)	DB		L1		L2		U1		CR		U2		AVE	
	A	B	A	B	A	B	A	B	A	B	A	B	A	B
1 (6)	25.0	1	31.65	0*	16.83	8	19.22	3	23.7	79	9.87	2636	21.06	454.5
2 (9)	36.68	9	48.17	14	24.27	163	22.64	29	35.91	937	12.97	10780#	30.11	1704.44
3 (12)	49.84	57	72.47	28	36.97	482	31.14	116	50.22	6102	17.69	10767#	43.06	2925.33
4 (9)	45.87	71	88.27	6	45.89	167	25.56	33	39.78	36861+	22.26	10786#	44.61	7987.33

A = unrep time in hours

B = CPU time in seconds (note that L1GS requires no optimization)

* = XA did 10 simplex iterations but reported 0 cpu time.

\+ = The cpu time limit for this problem was set at 100000 seconds because XA did not yield an *integer solution after 3 hours. Also, additional constraints were added to decrease problem size.*

\# = The solver was terminated after 3 hours of cpu time without achieving the 10% optimality criterion.

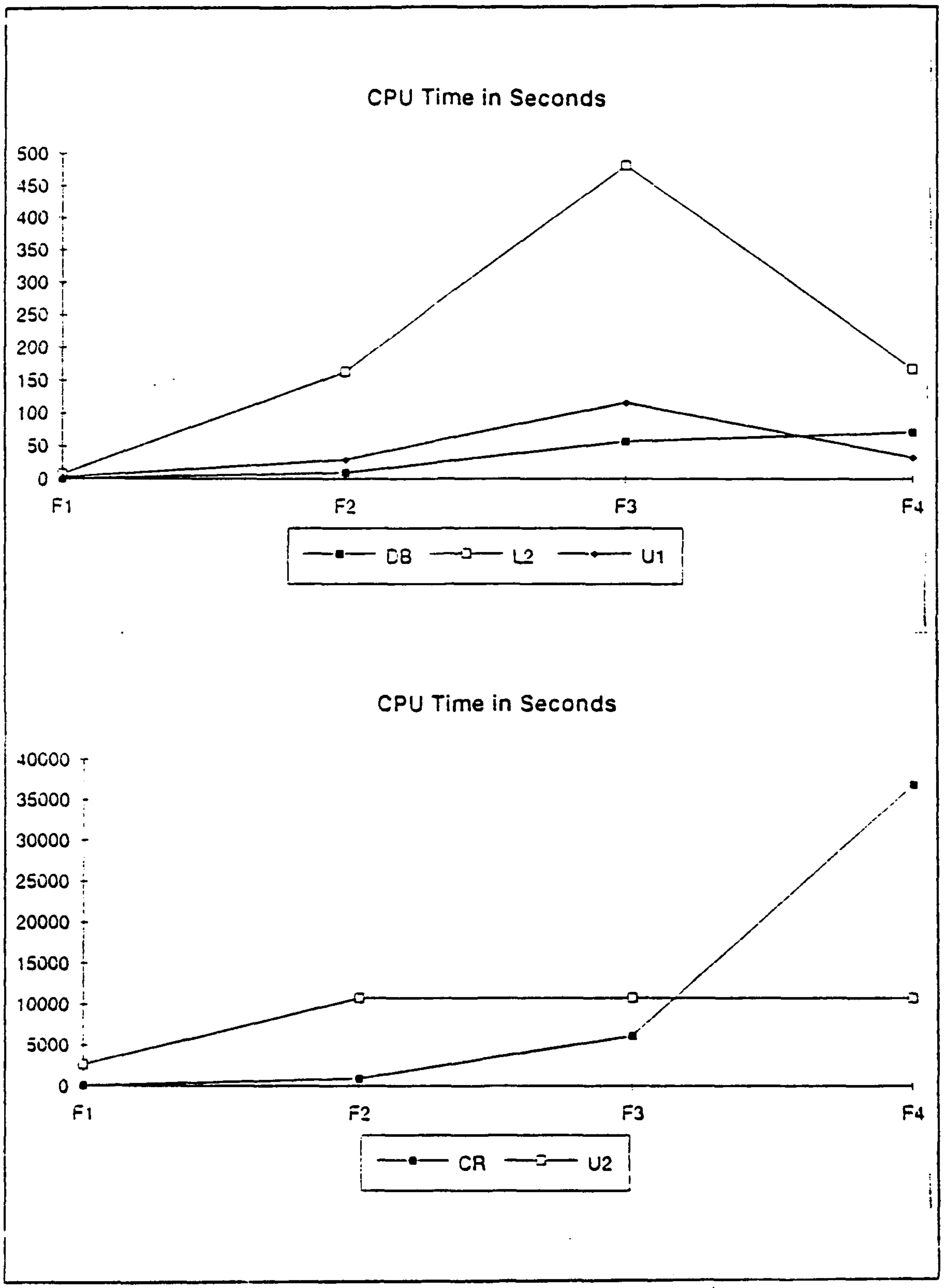


Figure 9 CPU times for the six possible tactics

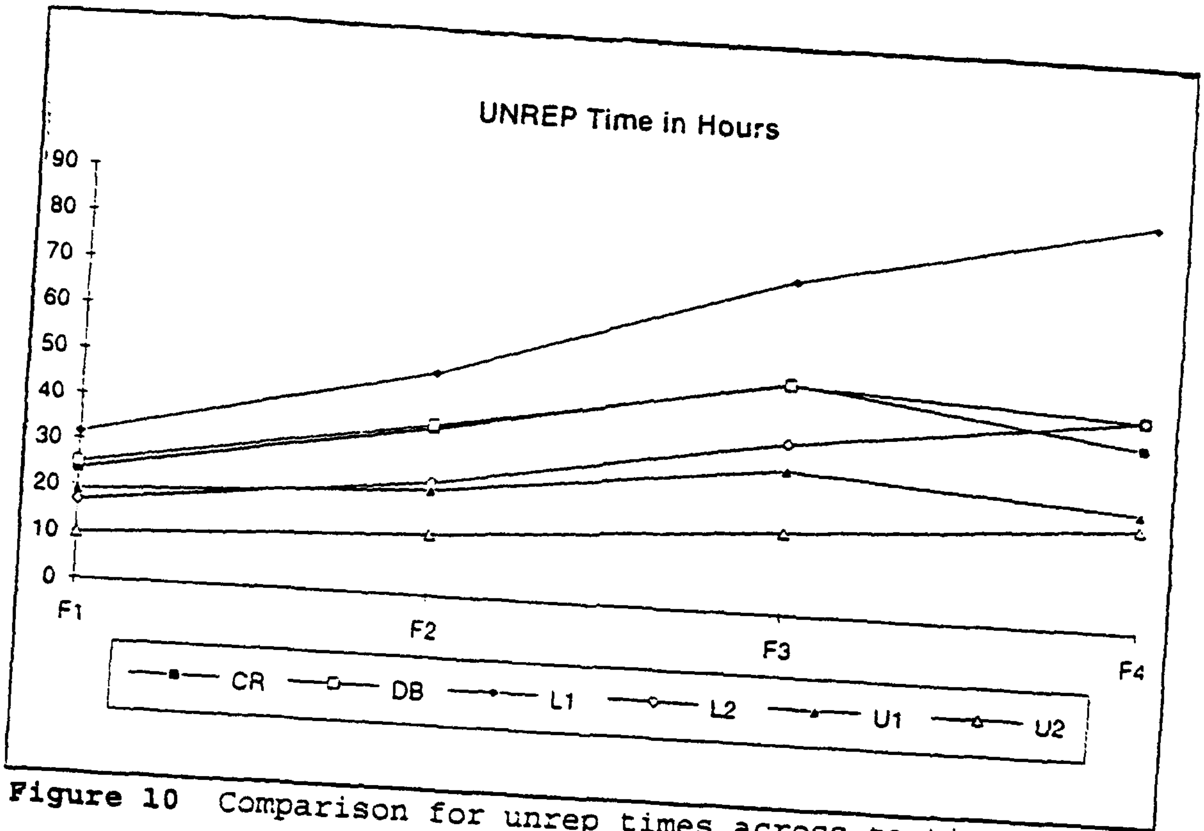


Figure 10 Comparison for unrep times across tactics

with the CR tactic are not solved to within 10% of optimality. For a more complete experimentation with a specialized algorithm for problems with the CR tactic, see Zabarouskas[1992].

B. REARMING SCENARIO

To demonstrate the algorithmic performance of the different models, the problem of maximizing combat values were solved with the time limits set at 50%, 75% and 85% of the minimum unrep time. It is important to note that the actual time limits are different from one tactic to the next. This is because different tactics yield different minimum unrep times. The results are summarized in Table 4 and 5. Figure 11 displays the average combat values

TABLE 4 COMBAT VALUE FOR REARMING MODELS

Combat values

	50 percent						75 percent					
Formation	CR	DB	L1	L2	U1	U2	CR	DB	L1	L2	U1	U2
1	22	22	28	28	22	18	29	28	33	33	33	28
2	31	31	33	33	28	26	38	42	42	40	39	39
3	40	39	47	43	40	38	NA	60	59	59	57	62
4	NA	33	39	40	30	33	39	44	47	44	42	44
Average	31	31	37	36	30	29	35	44	45	44	43	43

	85 PERCENT						MAX
Formation	CR	DB	L1	L2	U1	U2	
1	29	33	33	30	38	38	38
2	40	43	48	47	43	48	53
3	63	61	63	64	62	65	77
4	43	44	48	52	47	44	55
Average	44	45	48	48	48	49	56

TABLE 5 CPU TIME FOR REARMING MODELS

Computing Time in Seconds

Formation	50 percent CR	DB	L1	L2	U1	U2	75 percent CR	DB	L1	L2	U1	U2
1	10693	15	16	131	39	904	1637	65	10	121	10	1348
2	359	639	82	6806	265	9327	2246	221	126	506	394	352
3	10031*#	1558	105	2948#	1948	3126	NA	622	393	10792*	310	1838
4	NA	508	152	1287	793	10793*	6070	712	26	10792*#	1153	446
Average	7028	679	89	2793	761	6038	3318	405	139	5553	467	996

Formation	85 percent CR	DB	L1	L2	U1	U2
1	9	18	8	48	17	3489
2	129	91	249	1513	92	5451
3	10313	2823	427	10793*	1216	225
4	8449	10798*	61	2	457	10794*
Average	4725	3433	186	3089	446	4990

Note: * The solver was terminated after 3 hours of cpu time without achieving the 10% optimality criterion.
Additional constraints were added to decrease problem size.

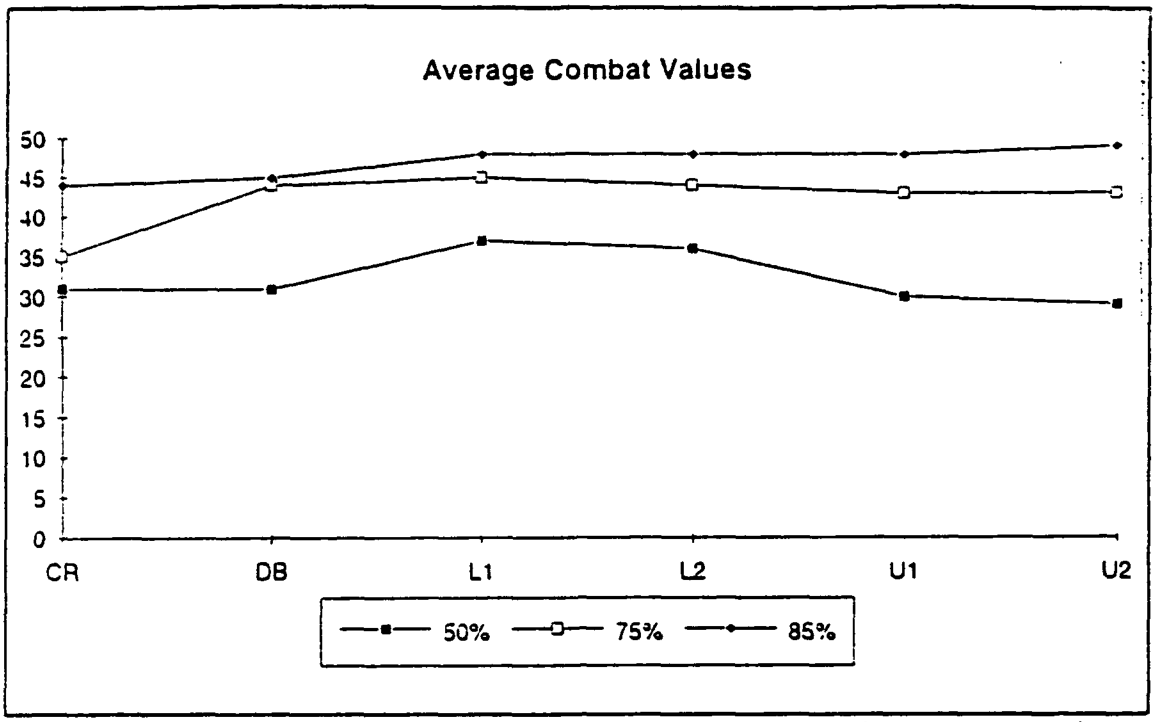


Figure 11 Average combat values over the four formations using the six tactics

over the four formations using the six tactics. Subject to the varied solution quality, the average combat values appeared to be constant across all six tactics. This confirms the intuition that, if the time limit is 50% of the minimum unrep time, the maximum combat value should be approximately 50% of the total combat value. In particular, the average combat value are 57.74%, 75.60% and 83.93% when the time limits are set to 50%, 75% and 85%, respectively. Figure 12 graphically shows the average cpu times require to solve the problems. Again, the tactics CR and U2 are among those requiring large amount of cpu time. To analyze the efficiency of the tactics, Figure 13 plots the maximum

combat values using formation 1 and time limits of 4.5, 6.25 and 7.65 hours. As expected, U2 achieves the highest combat value.

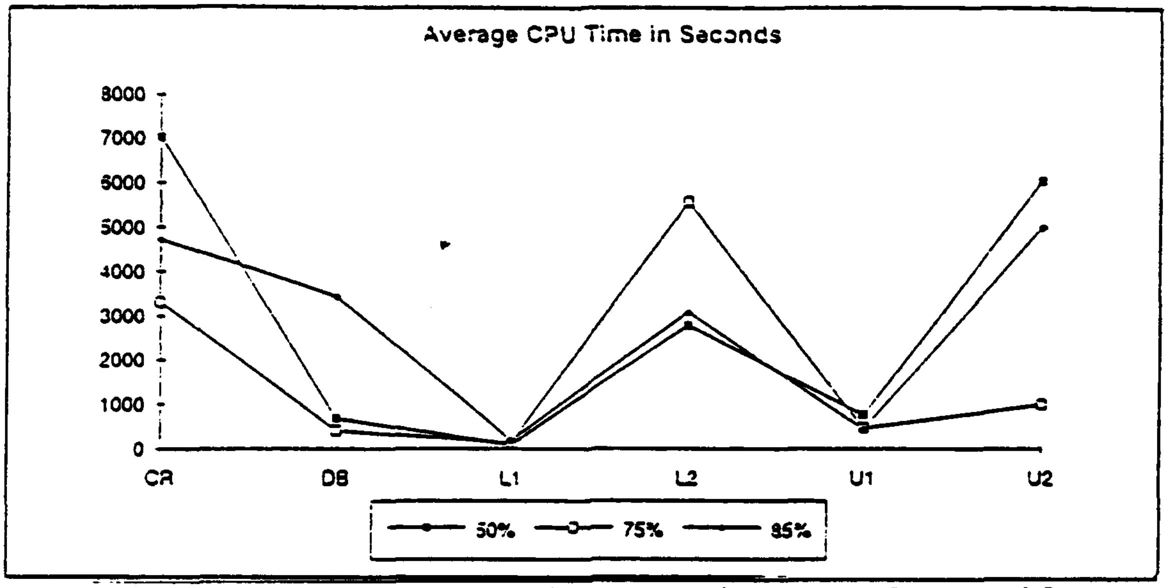


Figure 12 The average CPU times require to solve problems

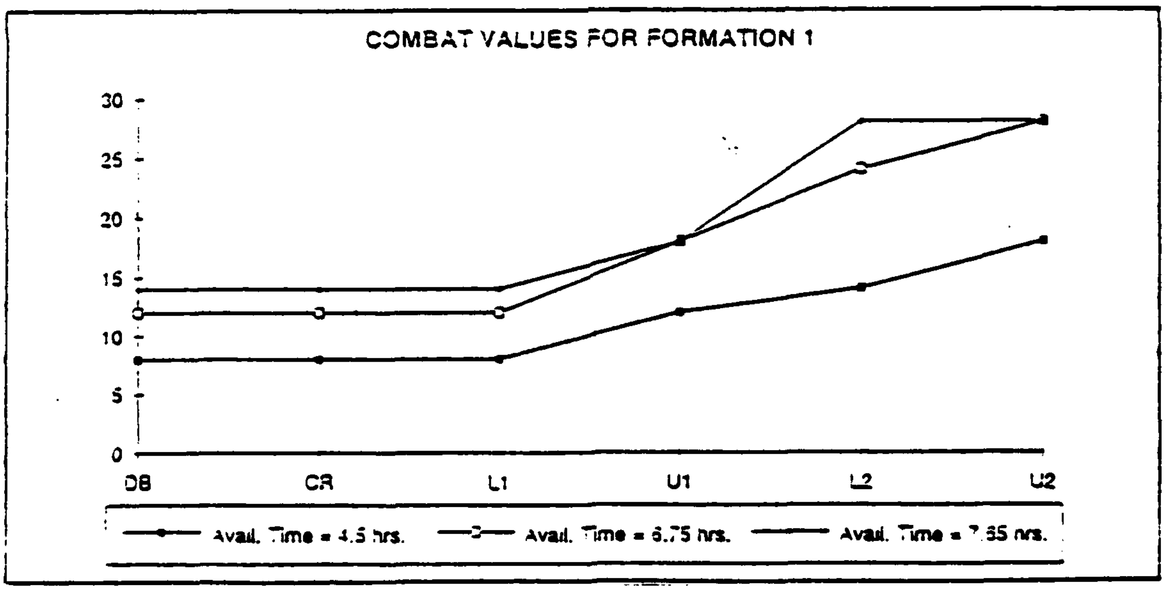


Figure 13 Maximum combat values using formation 1 and different time limit

IV. CONCLUSION

This thesis offers a classification of basic optimization models for planning underway replenishment of a battle group. In particular, this thesis focuses on two scenarios, *routine* and *rearming*, and considers three replenishment tactics: circuit rider, delivery boy and gas station. Optimization problems whose solutions yield plans for performing replenishment are developed. These problems all belong to an important class of problems in routing and scheduling called the traveling salesman problem. In the literature, researchers typically consider the traveling salesman problem and its generalizations such as generalized traveling salesman and prize collecting traveling salesman (or orienteering) problems with a large number of cities or locations. In contrast, problems for underway replenishment usually consist of less than 15 cities (ships) or group of cities (rendezvous locations). In addition, underway replenishment problems also offer new generalizations to the traveling salesman problem, e.g., the generalization of orienteering (maximize combat values problem with circuit rider strategy) and m-traveling salesmen problem (gas station tactic with two or more transfer stations).

Although many algorithms, optimal and heuristic, exist for the traveling salesman problem, they are developed with large scale applications in mind. To assist in the identification of fruitful areas for algorithmic development

particularly tailored toward underway replenishment, models developed in this thesis were implemented and solutions obtained using commercially available software such as GAMS and XA. The results show that replenishing with gas station with two transfer stations and circuit rider yield difficult optimization problems.

Being a preliminary study in the area of underway replenishment, this thesis points out many directions for future investigation; some of which are list below.

1) Develop models that take into account the fact that while replenishing the supply and combatant ships must travel at a speed slower than the formation speed for safety precautions.

2) Develop models which include stochastic components. In particular, parameters such as combat values and time available for replenishing depend on the incoming air raid whose axis and time are not known with certainty.

3) Develop specialized algorithms (optimal and/or heuristic) to solve difficult problems quickly.

APPENDIX A. ALTERNATE GAS STATION MODEL WITH LIMITED NUMBER OF SHIPS OFF-STATION RESTRICTION

Note: All decision variables and data used here are defined as before.

Minimum Unrep Time Problem

$$MIN \quad F$$

subject to

$$\sum_{i=1}^{n} (d_{ib}+r_i+d_{ei})\, V^a{}_i \leq F \,, \quad \forall\, a$$

$$\sum_{a=1}^{m} V^a{}_i = 1 \,, \quad \forall i$$

Maximum Combat Value Problem

$$MAX \quad \sum_{a=1}^{m} \sum_{i=1}^{n} w_i V^a{}_i$$

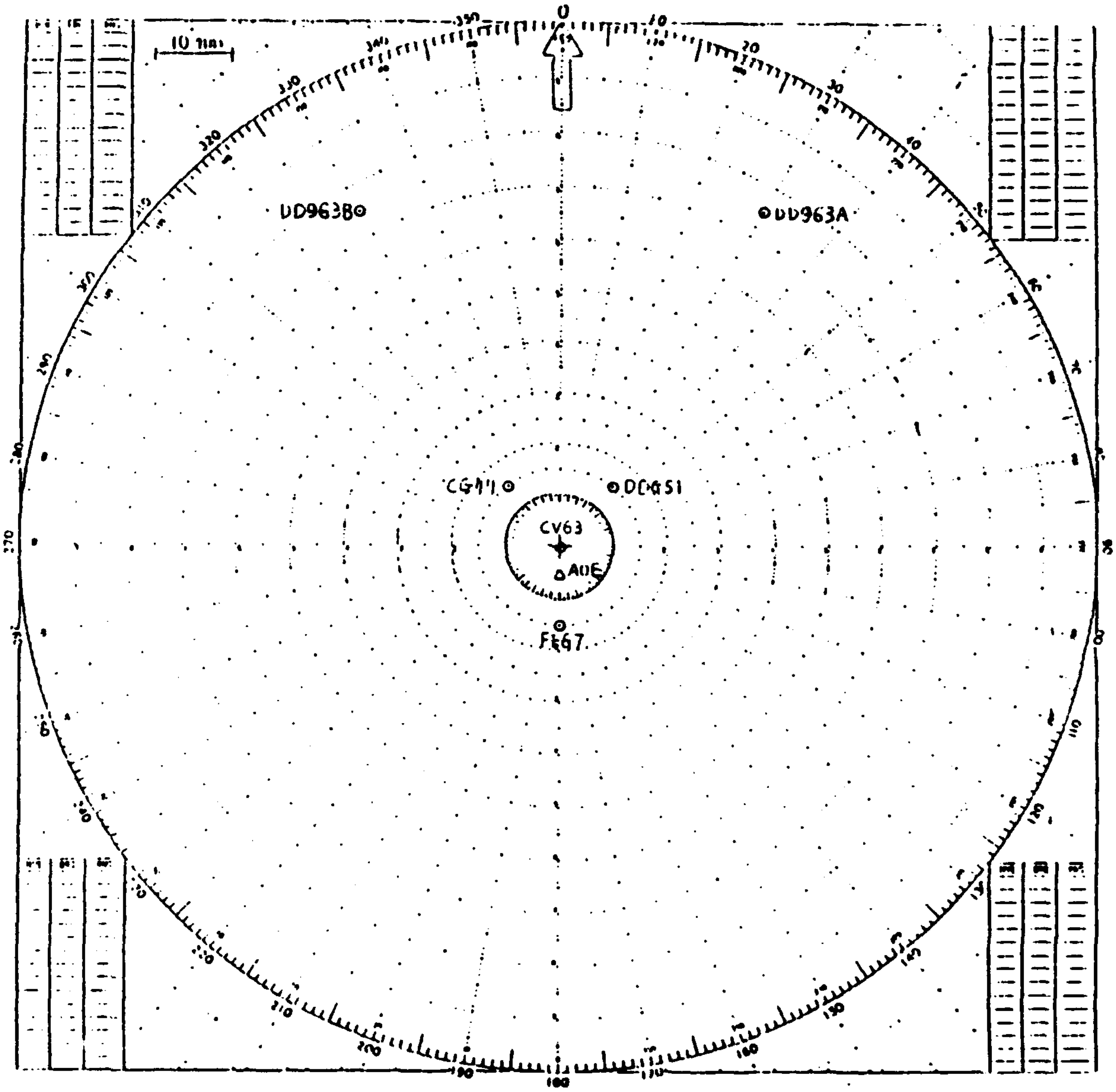
subject to

$$\sum_{i=1}^{n} (d_{ib}+r_i+d_{ei})\, V^a{}_i \leq h \,, \quad \forall a$$

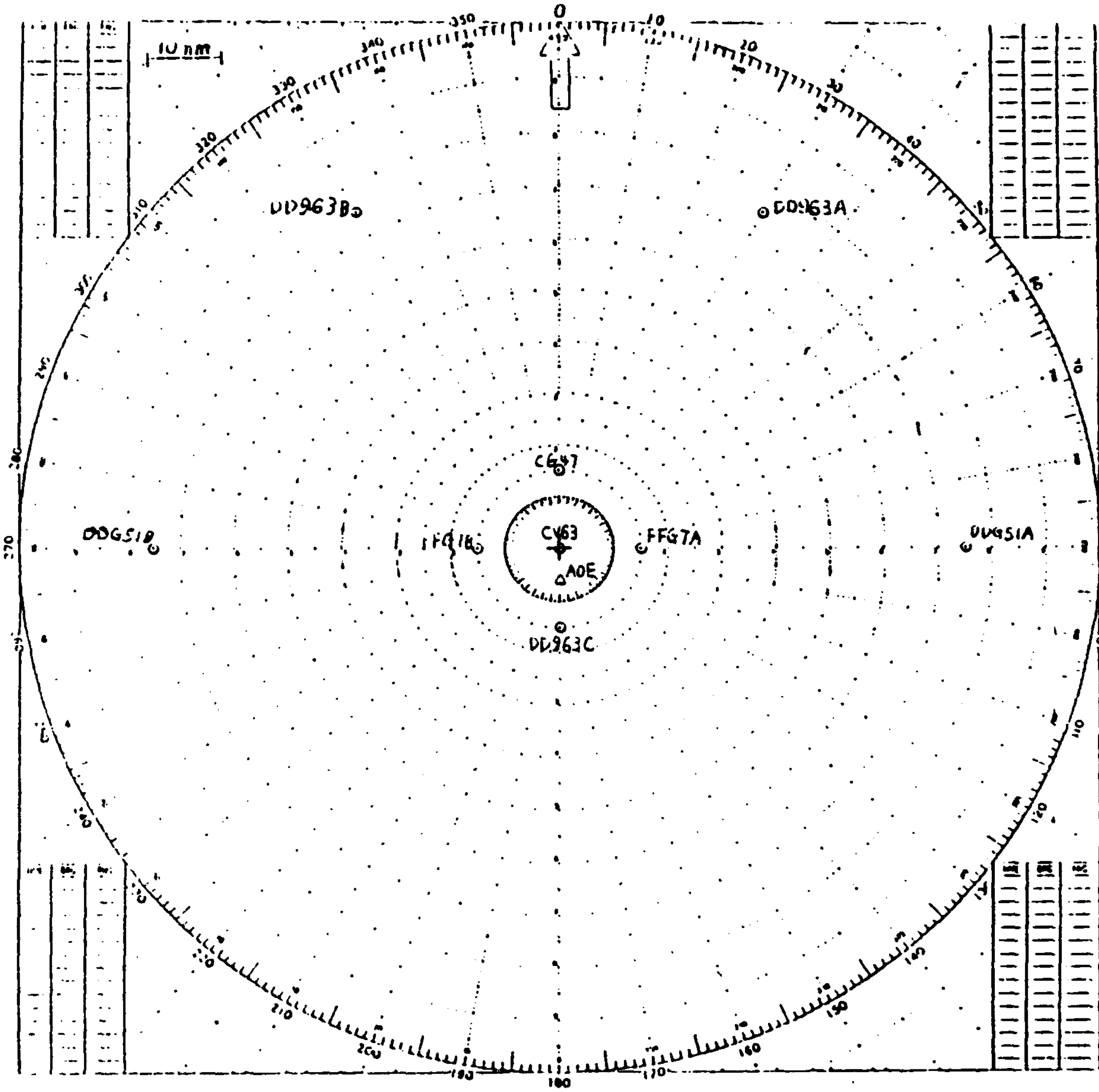
$$\sum_{a=1}^{m} V^a{}_i \leq 1 \,, \quad \forall i$$

APPENDIX B. FOUR BATTLE GROUPS FORMATION LAYOUT

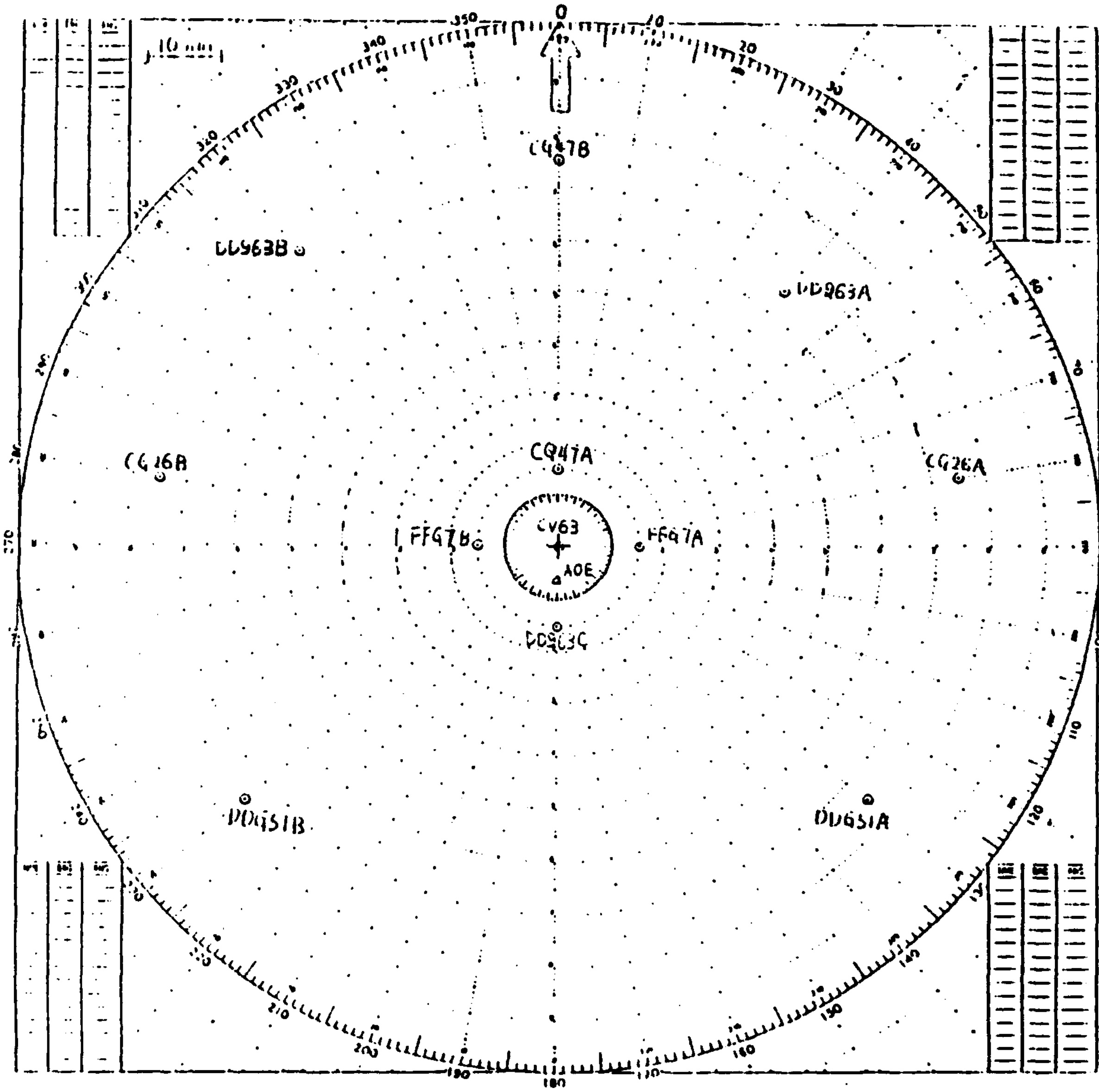
A. FORMATION 1



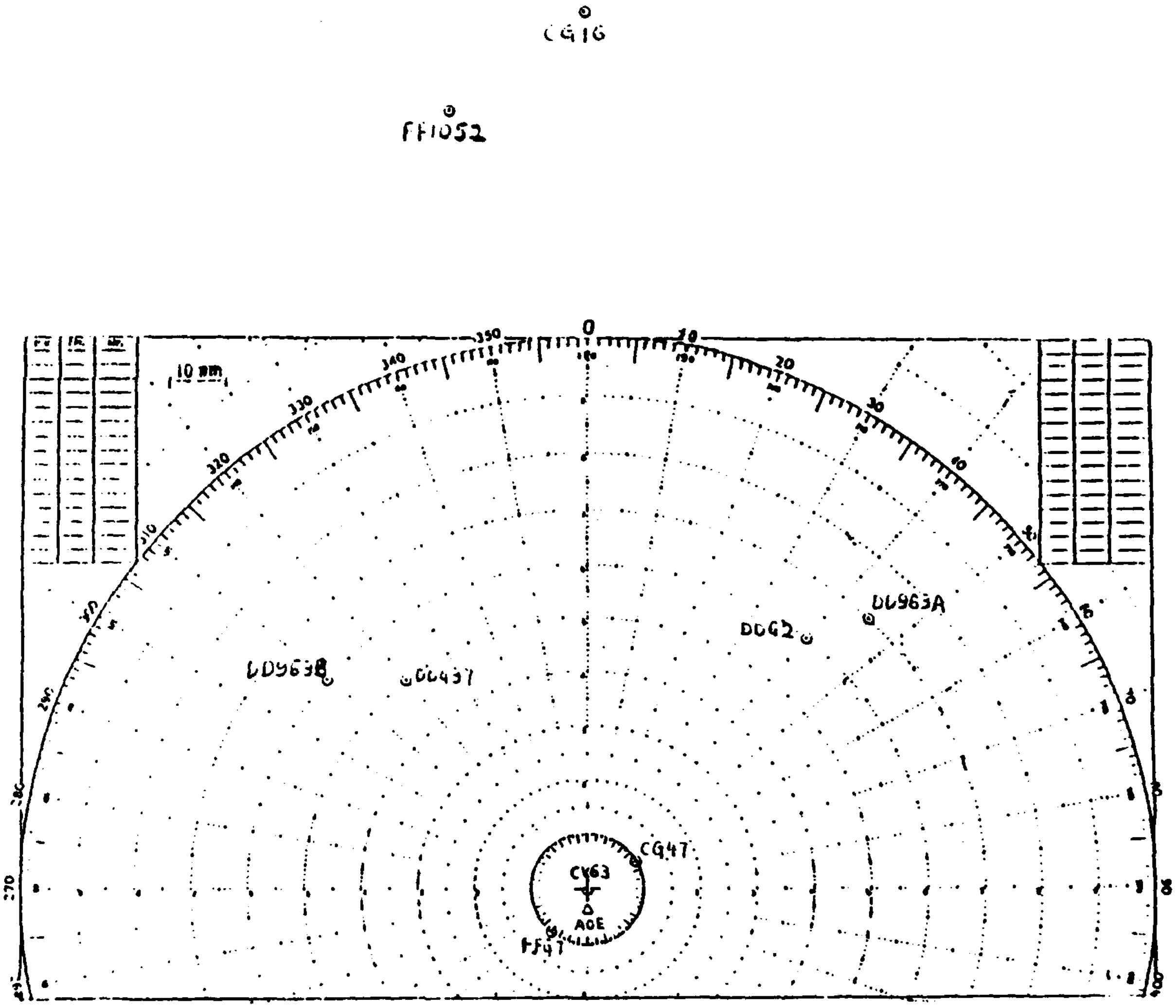
B. FORMATION 2



C. FORMATION 3



D. FORMATION 4



APPENDIX C. UNREP GAMS MODELS

A. DELIVERY BOY TACTIC WITH MINIMUM TOTAL UNREP TIME OBJECTIVE FUNCTION

```
$TITLE * * * Battle Group Replenishment Problem * * *
$STITLE * * * DELIVERY BOY TACTIC * * *
*-----------GAMS and dollar control options---------------------------
$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS LIMCOL = 0, LIMROW = 0,SOLPRINT = OFF, DECIMALS = 2;
OPTIONS RESLIM =10800,ITERLIM =1000000, OPTCR =
0.10,WORK=200000;
*OPTIONS MIP = ZOOM ;
 OPTIONS INTEGER1 = 6;

$INCLUDE 'BG-F1.DAT'
$INCLUDE 'SUB.SET'

*------------------COMPUTED DATA ------------------------------------
PARAMETER
  DIST(S,T)  travel time between candidate UNREP points;

  DIST(S,T)$(ORD(S) NE ORD(T) ) =
      (FP*(CP(T,'Y')-CP(S,'Y')) +
       SQRT(SQR(FP*(CP(T,'Y')-CP(S,'Y'))) +
       (SQR(SP)-SQR(FP))*(SQR(CP(T,'X')-CP(S,'X'))+
       SQR(CP(T,'Y')-CP(S,'Y')))))/(SQR(SP)-SQR(FP));

  DIST(S,BB) = 0;
  DIST(EE,T) = 0;
  DIST(S,T)$(ORD(S) EQ ORD(T)) = 0;

BINARY VARIABLE
  X(K,S,T)   EQUAL 1 IF ARC S P TO T Q IS SELECTED
                ;
 X.FX(K,S,T)$(ORD(S) EQ ORD(T)) = 0;
 X.FX(K,BB,EE) = 0;

VARIABLE
```

```
 TTIME;

EQUATIONS
  OBJ
  BEGIN
  FINISH
  BETWEEN(K,S)
  ONCE(S)
  ;

*                    > > >  OBJECTIVE FUNCTION < < <
OBJ..
 TTIME =E= SUM((BB,TT,FT), X(FT,BB,TT)*(DIST(BB,TT)+UTIME(TT)))
       +  SUM((TT,EE,LT), X(LT,TT,EE)*DIST(TT,EE))
       +  SUM((SS,TT,KK), X(KK,SS,TT)*(DIST(SS,TT)+UTIME(TT)));

*                    > > > subject to< < <
BEGIN..
 SUM((FT,BB,TT), X(FT,BB,TT)) =E= 1;

FINISH..
 SUM((LT,SS,EE), X(LT,SS,EE)) =E= 1;

BETWEEN(K,SS)$(ORD(K) GT 1)..

  SUM(BB,X(K-1,BB,SS))$(ORD(K) EQ 2) +
  SUM(TT,X(K-1,TT,SS))$(ORD(K) GT 2)    =E=
  SUM(TT,X(K,SS,TT))  $(ORD(K) LT CARD(K)) +
  SUM(EE,X(K,SS,EE))  $(ORD(K) EQ CARD(K))   ;

ONCE(SS)..
  SUM((FT,BB),X(FT,BB,SS)) + SUM((KK,TT),X(KK,TT,SS)) =E= 1;
*-----------------------------------------------------------------
MODEL UNREP  / ALL /;
SOLVE UNREP USING  MIP MINIMIZING TTIME;

*--------------------Solution Report-------------------------------

OPTION X:1:2:1;
DISPLAY TTIME.L, DIST, X.L;
```

B. DELIVERY BOY TACTIC WITH MAXIMUM COMBAT VALUE OBJECTIVE FUNCTION

```
$TITLE * * * Battle Group Replenishment Problem * * *
$STITLE * * * DELIVERY BOY TACTIC * * *
*-----------GAMS and dollar control options-------------------------
$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS LIMCOL = 0, LIMROW = 0,SOLPRINT = OFF, DECIMALS = 2;
OPTIONS RESLIM =10800,ITERLIM =1000000, OPTCR =
0.10,WORK=200000;
*OPTIONS MIP = ZOOM;
OPTIONS INTEGER1 = 6;

$INCLUDE 'BG-F1.DAT'

$INCLUDE 'SUB.SET'

*-----------------COMPUTED DATA ----------------------------------
PARAMETER
   DIST(S,T)  travel time between candidate UNREP points;

   DIST(S,T)$(ORD(S) NE ORD(T) ) =
       (FP*(CP(T,'Y')-CP(S,'Y')) +
        SQRT(SQR(FP*(CP(T,'Y')-CP(S,'Y'))) +
        (SQR(SP)-SQR(FP))*(SQR(CP(T,'X')-CP(S,'X'))+
        SQR(CP(T,'Y')-CP(S,'Y')))))/(SQR(SP)-SQR(FP));

   DIST(S,BB) = 0;
   DIST(EE,T) = 0;
   DIST(S,T)$(ORD(S) EQ ORD(T)) = 0;

BINARY VARIABLE
   X(K,S,T)    equal 1 if arc s to t is selected
   V(S)        equal 1 if ship s is visited
               ;
  X.FX(K,S,T)$(ORD(S) EQ ORD(T)) = 0;
  X.FX(K,BB,EE) = 0;
  X.FX(FT,ST,T) = 0;

VARIABLE
  CVAL;
```

```
EQUATIONS
   OBJ
   BEGIN
   FINISH
   BETWEEN(S,K)
   ONCE(S)
   TLIMIT
   ;

*                        > > >  OBJECTIVE FUNCTION < < <
OBJ..
 CVAL  =E= SUM(SS, VAL(SS)*V(SS));

*                        > > > subject to< < <
BEGIN..
 SUM((FT,BB,TT), X(FT,BB,TT))  =E=  1;

FINISH..
 SUM((KT,SS,EE), X(KT,SS,EE))  =E=  1;

BETWEEN(SS,K)$(ORD(K) GT 1)..
  SUM(BB,X(K-1,BB,SS))$(ORD(K) EQ 2)         +
  SUM(TT,X(K-1,TT,SS))$(ORD(K) GT 2)         =E=
  SUM(ST,X(K,SS,ST))  $(ORD(K) LT CARD(K))   +
  SUM(EE,X(K,SS,EE))  $(ORD(K) EQ CARD(K))   ;

ONCE(SS)..
  SUM((FT,BB),X(FT,BB,SS)) + SUM((KK,TT),X(KK,TT,SS))  =E=  V(SS);

TLIMIT..
  SUM(SS,UTIME(SS)*V(SS))                    +
  SUM((FT,BB,SS),DIST(BB,SS)*X(FT,BB,SS))    +
  SUM((KK,SS,ST),DIST(SS,ST)*X(KK,SS,ST))    +
  SUM((LT,SS,EE),DIST(SS,EE)*X(LT,SS,EE))   =L=  AVAIL;

*-------------------------------------------------------------------
MODEL UNREP  / ALL /;
SOLVE UNREP USING  MIP MAXIMIZING CVAL;

*-------------------Solution Report------------------------------

OPTION X:1:2:1;
DISPLAY CVAL.L, AVAIL, DIST, X.L;
```

C. CIRCUIT RIDER TACTIC WITH MINIMUM TOTAL UNREP TIME OBJECTIVE FUNCTION

```
$TITLE * * * Battle Group Replenishment Problem * * *
$STITLE * * * CIRCUIT RIDER TACTIC * * *
*-----------GAMS and dollar control options--------------------------
$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS LIMCOL = 0, LIMROW = 0,SOLPRINT = OFF, DECIMALS = 2;
OPTIONS RESLIM =10800,ITERLIM =1000000, OPTCR =
0.10,WORK=200000;
*OPTIONS MIP = ZOOM;
OPTIONS INTEGER1 = 6;

$INCLUDE 'BG-F1C.DAT'

$INCLUDE 'CSUB.SET'

*-----------------COMPUTED DATA -----------------------------------
PARAMETER
  DIST(S,P,T,Q)  travel time between candidate UNREP points;

  DIST(S,P,T,Q)$(ORD(S) NE ORD(T) ) =
      (FP*(CP(T,Q,'Y')-CP(S,P,'Y')) +
       SQRT(SQR(FP*(CP(T,Q,'Y')-CP(S,P,'Y'))) +
       (SQR(SP)-SQR(FP))*(SQR(CP(T,Q,'X')-CP(S,P,'X'))+
       SQR(CP(T,Q,'Y')-CP(S,P,'Y')))))/(SQR(SP)-SQR(FP));

  DIST(S,P,BB,P) = 0;
  DIST(EE,Q,T,Q) = 0;
  DIST(S,P,T,Q)$(ORD(S) EQ ORD(T)) = 0;

BINARY VARIABLE
  X(K,S,P,T,Q)    EQUAL 1 IF ARC S P TO T Q IS SELECTED
  V(S,P)
          ;
 X.FX(K,S,P,T,Q)$(ORD(S) EQ ORD(T)) = 0;
 V.FX(BB,BP) = 1;
 V.FX(EE,EP) = 1;

VARIABLE
 TTIME;
```

```
EQUATIONS
  OBJ
  BEGIN
  FINISH
  BETWEEN(K,S,P)
  ONCE(S,P)
  VISIT(S)
      ;
*                    > > >  OBJECTIVE FUNCTION < < <
OBJ..
 TTIME =E= SUM((FT,BB,BP,TT,Q),
            X(FT,BB,BP,TT,Q)*(DIST(BB,BP,TT,Q)+UTIME(TT)))
       +  SUM((LT,TT,Q,EE,EP), X(LT,TT,Q,EE,EP)*DIST(TT,Q,EE,EP))
       +  SUM((KK,SS,P,TT,Q),
            X(KK,SS,P,TT,Q)*(DIST(SS,P,TT,Q)+UTIME(TT)));

*                    > > > subject to< < <
BEGIN..
 SUM((FT,BB,BP,TT,Q), X(FT,BB,BP,TT,Q)) =E= 1;

FINISH..
 SUM((LT,SS,P,EE,EP), X(LT,SS,P,EE,EP)) =E= 1;

BETWEEN(K,SS,P)$(ORD(K) GT 1)..
  SUM((BB,BP),X(K-1,BB,BP,SS,P))$(ORD(K) EQ 2) +
  SUM((TT,Q),X(K-1,TT,Q,SS,P))  $(ORD(K) GT 2)    =E=
  SUM((TT,Q),X(K,SS,P,TT,Q))    $(ORD(K) LT CARD(K)) +
  SUM((EE,EP),X(K,SS,P,EE,EP))  $(ORD(K) EQ CARD(K))   ;

ONCE(SS,P)..
  SUM((FT,BB,BP),X(FT,BB,BP,SS,P)) +
  SUM((KK,TT,Q), X(KK,TT,Q,SS,P)) =E= V(SS,P);

VISIT(S)..
  SUM(P, V(S,P)) =E= 1;

*-----------------------------------------------------------------
MODEL UNREP  / ALL /;
SOLVE UNREP USING  MIP MINIMIZING TTIME;

*-------------------Solution Report------------------------------
OPTION X:1:3:2;
DISPLAY TTIME.L, DIST, X.L;
```

D. CIRCUIT RIDER TACTIC WITH MAXIMUM COMBAT VALUE OBJECTIVE FUNCTION

```
$TITLE * * * Battle Group Replenishment Problem * * *
$STITLE * * * CIRCUIT RIDER TACTIC * * *
*-----------GAMS and dollar control options--------------------------
$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS LIMCOL = 0, LIMROW =  0,SOLPRINT = OFF, DECIMALS = 2;
OPTIONS RESLIM =108000,ITERLIM =1000000, OPTCR =
0.10,WORK=200000;
*OPTIONS MIP = ZOOM;
OPTIONS INTEGER1 = 6

$INCLUDE 'BG-F1C.DAT'

$INCLUDE 'CSUB.SET'

*------------------COMPUTED DATA -----------------------------------
PARAMETER
  DIST(S,P,T,Q)  travel time between candidate UNREP points;

  DIST(S,P,T,Q)$(ORD(S) NE ORD(T) ) =
      (FP*(CP(T,Q,'Y')-CP(S,P,'Y')) +
       SQRT(SQR(FP*(CP(T,Q,'Y')-CP(S,P,'Y'))) +
       (SQR(SP)-SQR(FP))*(SQR(CP(T,Q,'X')-CP(S,P,'X'))+
       SQR(CP(T,Q,'Y')-CP(S,P,'Y')))))/(SQR(SP)-SQR(FP));

  DIST(S,P,BB,P) = 0;
  DIST(EE,Q,T,Q) = 0;
  DIST(S,P,T,Q)$(ORD(S) EQ ORD(T)) = 0;

BINARY VARIABLE
  X(K,S,P,T,Q)    EQUAL 1 IF ARC S P TO T Q IS SELECTED
  V(S,P)
          ;
 X.FX(K,S,P,T,Q)$(ORD(S) EQ ORD(T)) = 0;

VARIABLE
 CVAL;

EQUATIONS
```

```
   OBJ
   BEGIN
   FINISH
   BETWEEN(K,S,P)
   ONCE(S,P)
   VISIT(S)
   TLIMIT    ;

*                  > > > OBJECTIVE FUNCTION < < <
OBJ..
 CVAL =E= SUM((SS,P), VAL(SS)*V(SS,P));

*                  > > > subject to< < <
BEGIN..
 SUM((FT,BB,BP,TT,Q), X(FT,BB,BP,TT,Q)) =E= 1;

FINISH..
 SUM((KT,SS,P,EE,EP), X(KT,SS,P,EE,EP)) =E= 1;

BETWEEN(K,SS,P)$(ORD(K) GT 1)..
  SUM((BB,BP),X(K-1,BB,BP,SS,P))$(ORD(K) EQ 2)      +
  SUM((TT,Q),X(K-1,TT,Q,SS,P))  $(ORD(K) GT 2)      =E=
  SUM((ST,Q),X(K,SS,P,ST,Q))    $(ORD(K) LT CARD(K)) +
  SUM((EE,EP),X(K,SS,P,EE,EP))  $(ORD(K) EQ CARD(K))   ;

ONCE(SS,P)..
  SUM((FT,BB,BP),X(FT,BB,BP,SS,P)) + SUM((KK,TT,Q),X(KK,TT,Q,SS,P))
                                     =E= V(SS,P);

VISIT(SS)..
  SUM(P,V(SS,P)) =L= 1;

TLIMIT..
  SUM((SS,P),UTIME(SS)*V(SS,P))                          +
  SUM((FT,BB,BP,SS,P),DIST(BB,BP,SS,P)*X(FT,BB,BP,SS,P))   +
  SUM((KK,SS,P,ST,Q),DIST(SS,P,ST,Q)*X(KK,SS,P,ST,Q))      +
  SUM((LT,SS,P,EE,EP),DIST(SS,P,EE,EP)*X(LT,SS,P,EE,EP)) =L= AVAIL;
*----------------------------------------------------------------
MODEL UNREP  / ALL /;
SOLVE UNREP USING  MIP MAXIMIZING CVAL;
*-------------------Solution Report-----------------------------
OPTION X:1:3:2;
DISPLAY CVAL.L,DIST,AVAIL, X.L;
```

E. GAS STATION TACTIC WITH MINIMUM TOTAL UNREP TIME OBJECTIVE FUNCTION, TWO TRANSFER STATIONS AND LIMITED NUMBER OF SHIPS OFF-STATION

```
$TITLE * * * Battle Group Replenishment Problem * * *
$STITLE * * * GAS STATION TACTIC * * *
*----------GAMS and dollar control options------------------------
$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS LIMCOL = 0, LIMROW =  0,SOLPRINT = OFF, DECIMALS = 2;
OPTIONS  RESLIM  =10800,ITERLIM  =1000000,  OPTCR  =
0.10,WORK=200000;
*OPTIONS MIP = ZOOM;
 OPTIONS INTEGER1 = 6;

$INCLUDE 'BGF-2.DAT'

$INCLUDE 'SUB.SET'

SET
  A       supply ships          /A1, A2/
          ;

ALIAS (SS,TT);

*----------------COMPUTED DATA ------------------------------------
PARAMETER
  DIST(S,T)  travel time between candidate UNREP points;

  DIST(S,T)$(ORD(S) NE ORD(T) ) =
      (FP*(CP(T,'Y')-CP(S,'Y')) +
       SQRT(SQR(FP*(CP(T,'Y')-CP(S,'Y'))) +
       (SQR(SP)-SQR(FP))*(SQR(CP(T,'X')-CP(S,'X'))+
       SQR(CP(T,'Y')-CP(S,'Y')))))/(SQR(SP)-SQR(FP));

  DIST(S,T)$(ORD(S) EQ ORD(T)) = 0;

*----------------MODEL----------------------------------------
BINARY VARIABLE
  X(A,K,S,T)    equal 1 if arc x is selected to be sequence k by a
          ;
 X.FX(A,K,S,T)$(ORD(S) EQ ORD(T)) = 0;
```

```
POSITIVE VARIABLE
  TMAX;

VARIABLE
  TTIME;

EQUATIONS
   OBJ
   MAXT(A)
   BEGIN(A)
   FINISH(A)
   BETWEEN(A,K,S)
   ONCE(S)
   ;

*                   > > > OBJECTIVE FUNCTION < < <
OBJ..
 TTIME   =E=  TMAX;

*                   > > > subject to< < <
MAXT(A)..
TMAX  =G=  SUM((FT,BB,SS), X(A,FT,BB,SS)*(DIST(SS,BB)+UTIME(SS)))
       +  SUM((LT,TT,EE), X(A,LT,TT,EE)*DIST(EE,TT))
       +  SUM((KK,SS,ST), X(A,KK,SS,ST)
       * SUM((BB,EE), DIST(EE,SS)+UTIME(ST)+DIST(ST,BB))) ;

BEGIN(A)..
 SUM((FT,BB,TT), X(A,FT,BB,TT)) =E= 1;

FINISH(A)..
 SUM((KT,SS,EE), X(A,KT,SS,EE)) =E= 1;

BETWEEN(A,K,SS)$(ORD(K) GT 1)..
  SUM(BB,X(A,K-1,BB,SS))$(ORD(K) EQ 2)      +
  SUM(TT,X(A,K-1,TT,SS))$(ORD(K) GT 2)      =E=
  SUM(ST,X(A,K,SS,ST))  $(ORD(K) LT CARD(K)) +
  SUM(EE,X(A,K,SS,EE))  $(ORD(K) EQ CARD(K)) ;

ONCE(SS)..
  SUM(A,SUM((FT,BB),X(A,FT,BB,SS)) +
  SUM((KK,TT),X(A,KK,TT,SS)))      =E= 1;

*-----------------------------------------------------------------
```

```
MODEL UNREP  / ALL /;
SOLVE UNREP USING  MIP MINIMIZING TTIME;

*------------------Solution Report----------------------------

OPTION X:1:2:1;
DISPLAY TTIME.L, DIST, X.L;
```

F. GAS STATION TACTIC WITH MAXIMUM COMBAT VALUE OBJECTIVE FUNCTION, TWO TRANSFER STATIONS AND LIMITED NUMBER OF SHIPS OFF-STATION

```
$TITLE * * * Battle Group Replenishment Problem * * *
$STITLE * * * GAS STATION TACTIC * * *
*-----------GAMS and dollar control options-------------------------
$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS LIMCOL = 0, LIMROW = 0,SOLPRINT = OFF, DECIMALS = 2;
OPTIONS RESLIM = 10800,ITERLIM =1000000, OPTCR =
0.10,WORK=200000;
*OPTIONS MIP = ZOOM;
OPTIONS INTEGER1 = 6;

$INCLUDE 'BGF-1.DAT'

$INCLUDE 'SUB.SET'

SET
  A       supply ships        /A1, A2/ ;

SCALAR
  AVAIL    available unrep time /8.42/;


*-----------------COMPUTED DATA -----------------------------------
PARAMETER
  DIST(S,T)  travel time between candidate UNREP points;

  DIST(S,T)$(ORD(S) NE ORD(T) ) =
      (FP*(CP(T,'Y')-CP(S,'Y')) +
       SQRT(SQR(FP*(CP(T,'Y')-CP(S,'Y'))) +
       (SQR(SP)-SQR(FP))*(SQR(CP(T,'X')-CP(S,'X'))+
       SQR(CP(T,'Y')-CP(S,'Y')))))/(SQR(SP)-SQR(FP));

  DIST(S,T)$(ORD(S) EQ ORD(T)) = 0;

*------------------MODEL------------------------------------------

BINARY VARIABLE
  X(A,K,S,T)    equal 1 if arc x is selected to be sequence k by a
```

```
  V(A,S)       equal 1 if ship s is replenished at station a
              ;
 X.FX(A,K,S,T)$(ORD(S) EQ ORD(T)) = 0;

VARIABLE
 CVAL;

EQUATIONS
  OBJ
  BEGIN(A)
  FINISH(A)
  BETWEEN(A,K,S)
  ONCE(S)
  ONCE1(S)
  TLIMIT(A)
  ;

*                    > > >  OBJECTIVE FUNCTION < < <
OBJ..
 CVAL =E= SUM((A,SS), VAL(SS)*V(A,SS));

*                    > > > subject to< < <
BEGIN(A)..
 SUM((FT,BB,TT), X(A,FT,BB,TT)) =E= 1;

FINISH(A)..
 SUM((KT,SS,EE), X(A,KT,SS,EE)) =E= 1;

BETWEEN(A,K,SS)$(ORD(K) GT 1)..
  SUM(BB,X(A,K-1,BB,SS))$(ORD(K) EQ 2) +
  SUM(TT,X(A,K-1,TT,SS))$(ORD(K) GT 2)    =E=
  SUM(ST,X(A,K,SS,ST)) $(ORD(K) LT CARD(K)) +
  SUM(EE,X(A,K,SS,EE)) $(ORD(K) EQ CARD(K))   ;

ONCE(SS)..
  SUM(A,SUM((FT,BB),X(A,FT,BB,SS)) +
  SUM((KK,TT),X(A,KK,TT,SS)))      =E= SUM(A,V(A,SS));

ONCE1(SS)..
  SUM(A,V(A,SS)) =L= 1;

TLIMIT(A)..
  SUM(SS,UTIME(SS)*V(A,SS))                    +
```

```
 SUM((FT,BB,SS),DIST(SS,BB)*X(A,FT,BB,SS))       +
 SUM((KK,SS,ST),SUM((BB,EE), DIST(EE,SS)         +
          DIST(ST,BB))*X(A,KK,SS,ST)) +
 SUM((LT,SS,EE),DIST(EE,SS)*X(A,LT,SS,EE))      =L= AVAIL;

*-----------------------------------------------------------------
MODEL UNREP  / ALL /;
SOLVE UNREP USING  MIP MAXIMIZING CVAL;

*--------------------Solution Report--------------------------------

OPTION X:1:2:1;
DISPLAY CVAL.L,DIST,AVAIL, X.L;
```

G. GAS STATION TACTIC WITH MINIMUM TOTAL UNREP TIME OBJECTIVE FUNCTION, TWO TRANSFER STATIONS AND UNLIMITED NUMBER OF SHIPS OFF-STATION

```
$TITLE * * * Battle Group Replenishment Problem * * *
$STITLE * * * GAS STATION TACTIC * * *
*-----------GAMS and dollar control options--------------------------
$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS LIMCOL = 0, LIMROW =  0,SOLPRINT = OFF, DECIMALS = 2;
OPTIONS  RESLIM  =10800,ITERLIM  =1000000,  OPTCR  =
0.10,WORK=200000;
*OPTIONS MIP = ZOOM;
 OPTIONS INTEGER1 = 6;

SET
   A     unrep station /A1,A2/;

$INCLUDE 'BGF-1.DAT'

$INCLUDE 'SUB.SET'

ALIAS (K,L);

*-----------------COMPUTED DATA -----------------------------------
PARAMETER
  DIST(S,T)  travel time between candidate UNREP points;

  DIST(S,T)$(ORD(S) NE ORD(T) ) =
      (FP*(CP(T,'Y')-CP(S,'Y')) +
       SQRT(SQR(FP*(CP(T,'Y')-CP(S,'Y'))) +
       (SQR(SP)-SQR(FP))*(SQR(CP(T,'X')-CP(S,'X'))+
       SQR(CP(T,'Y')-CP(S,'Y')))))/(SQR(SP)-SQR(FP));

  DIST(S,T)$(ORD(S) EQ ORD(T)) = 0;

*------------------MODEL------------------------------------------
BINARY VARIABLE
  X(A,K,S,T)    equal 1 if arc x is selected to be sequence k by a
              ;
 X.FX(A,K,S,T)$(ORD(S) EQ ORD(T)) = 0;
```

```
POSITIVE VARIABLE
 GAP(A,K)
 TFINISH ;

NEGATIVE VARIABLE
 TSTART;

VARIABLE
 TTIME;

EQUATIONS
  OBJ
  START(A,K)
  FINISH(A,K)
  BEGIN(A)
  END(A)
  BETWEEN(A,K,S)
  ONCE(S)
  ;

*                  > > >  OBJECTIVE FUNCTION < < <
OBJ..
 TTIME  =E= TFINISH - TSTART ;

*                  > > > subject to< < <
START(A,K)$(ORD(K) LT CARD(K))..
 TSTART =L= (-SUM((FT,BB,TT),
            X(A,FT,BB,TT)*DIST(TT,BB))+GAP(A,K))$(ORD(K) EQ 1)
       + (SUM((FT,BB,TT),
             X(A,FT,BB,TT)*UTIME(TT)) + GAP(A,'1')
       +  SUM(L$(ORD(L) GT 1 AND ORD(L) LT ORD(K)),
             SUM((SS,TT),
                X(A,L,SS,TT)*UTIME(TT))
            + GAP(A,L))
       - SUM((SS,TT,BB),
             X(A,K,SS,TT)*DIST(TT,BB))+GAP(A,K))$(ORD(K) GT 1);

FINISH(A,K)$(ORD(K) LT CARD(K))..
 TFINISH =G= (SUM((FT,BB,TT),
            X(A,FT,BB,TT)*(DIST(BB,TT)+UTIME(TT)))
            + GAP(A,K))$(ORD(K) EQ 1)
       + (SUM((FT,BB,TT),
             X(A,FT,BB,TT)*UTIME(TT)) + GAP(A,'1')
```

```
        + SUM(L$(ORD(L) GT 1 AND ORD(L) LT ORD(K)),
            SUM((SS,TT),
                X(A,L,SS,TT)*UTIME(TT))
           + GAP(A,L))
        + SUM((SS,TT,EE),
            X(A,K,SS,TT)*(DIST(EE,TT)+UTIME(TT)))+GAP(A,K))
         $(ORD(K) GT 1);
*               > > > subject to< < <
BEGIN(A)..
 SUM((FT,BB,TT), X(A,FT,BB,TT)) =E= 1;

END(A)..
 SUM((KT,SS,EE), X(A,KT,SS,EE)) =E= 1;

BETWEEN(A,K,SS)$(ORD(K) GT 1)..
  SUM(BB,X(A,K-1,BB,SS))$(ORD(K) EQ 2)      +
  SUM(TT,X(A,K-1,TT,SS))$(ORD(K) GT 2)      =E=
  SUM(ST,X(A,K,SS,ST))  $(ORD(K) LT CARD(K)) +
  SUM(EE,X(A,K,SS,EE))  $(ORD(K) EQ CARD(K))   ;

ONCE(SS)..
  SUM(A,SUM((FT,BB),X(A,FT,BB,SS)) + SUM((KK,TT),X(A,KK,TT,SS))) =E=
1;

*-----------------------------------------------------------------

MODEL UNREP  / ALL /;
SOLVE UNREP USING  MIP MINIMIZING TTIME;

*--------------------Solution Report--------------------------------

PARAMETER REPORT(A,*,K);
 REPORT(A,'LEAVE',K)$(ORD(K) LT CARD(K))
     =(- SUM((FT,BB,TT),
           X.L(A,FT,BB,TT)*DIST(TT,BB))+GAP.L(A,K))$(ORD(K) EQ 1)
      + (SUM((FT,BB,TT),
           X.L(A,FT,BB,TT)*UTIME(TT)) + GAP.L(A,'1')
      + SUM(L$(ORD(L) GT 1 AND ORD(L) LT  ORD(K)),
           SUM((SS,TT),
               X.L(A,L,SS,TT)*UTIME(TT))
            + GAP.L(A,L))
     - SUM((SS,TT,BB),
           X.L(A,K,SS,TT)*DIST(TT,BB))+GAP.L(A,K))$(ORD(K) GT 1);
```

```
REPORT(A,'ARRIVE',K)$(ORD(K) LT CARD(K) AND ORD(K) GT 1)
    = REPORT(A,'LEAVE',K) +
     SUM((S,T,BB),(DIST(T,BB)+GAP.L(A,K))*X.L(A,K,S,T)) ;

REPORT(A,'ARRIVE',K)$(ORD(K) EQ 1)
    = REPORT(A,'LEAVE',K) +
     SUM((S,T,BB),(DIST(T,BB))*X.L(A,K,S,T)) ;

REPORT(A,'DEPART',K)$(ORD(K) LT CARD(K))
    = REPORT(A,'ARRIVE',K) +
     SUM((S,T),UTIME(T)*X.L(A,K,S,T)) ;

REPORT(A,'RETURN',K)$(ORD(K) LT CARD(K))
    = (SUM((FT,BB,TT),
         X.L(A,FT,BB,TT)*(DIST(BB,TT)+UTIME(TT)))
         + GAP.L(A,K))$(ORD(K) EQ 1)
    + (SUM((FT,BB,TT),
         X.L(A,FT,BB,TT)*UTIME(TT)) + GAP.L(A,'1')
    + SUM(L$(ORD(L) GT 1 AND ORD(L) LT ORD(K)),
         SUM((SS,TT),
            X.L(A,L,SS,TT)*UTIME(TT))
         + GAP.L(A,L))
    + SUM((SS,TT,EE),
        X.L(A,K,SS,TT)*(DIST(EE,TT)+UTIME(TT)))+GAP.L(A,K))
        $(ORD(K) GT 1);

OPTION X:1:2:1;
OPTION REPORT:3:1:1;
DISPLAY TTIME.L, DIST, X.L;
DISPLAY TSTART.L, TFINISH.L;
DISPLAY REPORT, GAP.L;
```

H. GAS STATION TACTIC WITH MAXIMUM COMBAT VALUE OBJECTIVE FUNCTION, TWO TRANSFER STATIONS AND UNLIMITED NUMBER OF SHIPS OFF-STATION

```
$TITLE * * * Battle Group Replenishment Problem * * *
$STITLE * * * GAS STATION TACTIC * * *
*-----------GAMS and dollar control options--------------------------
$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS LIMCOL = 0, LIMROW =  0,SOLPRINT = OFF, DECIMALS = 2;
OPTIONS   RESLIM   =   10800,ITERLIM   =1000000,   OPTCR   =
0.10,WORK=200000;
*OPTIONS MIP = zoom;
 OPTIONS INTEGER1 = 6;

SET
  A       supply ships         /A1, A2/ ;

$INCLUDE 'BGF-1.DAT'

$INCLUDE 'SUB.SET'

ALIAS (K,L);

SCALAR
  AVAIL    available unrep time    /5.5/;

*-----------------COMPUTED DATA -----------------------------------
PARAMETER
  DIST(S,T)  travel time between candidate UNREP points;

  DIST(S,T)$(ORD(S) NE ORD(T) ) =
      (FP*(CP(T,'Y')-CP(S,'Y')) +
       SQRT(SQR(FP*(CP(T,'Y')-CP(S,'Y'))) +
       (SQR(SP)-SQR(FP))*(SQR(CP(T,'X')-CP(S,'X'))+
       SQR(CP(T,'Y')-CP(S,'Y')))))/(SQR(SP)-SQR(FP));

  DIST(S,T)$(ORD(S) EQ ORD(T)) = 0;

*------------------MODEL-----------------------------------------
BINARY VARIABLE
  X(A,K,S,T)    equal 1 if arc x is selected to be sequence k by a
```

```
  V(A,S)        equal 1 if ship s is to be replenished at station a
          ;
 X.FX(A,K,S,T)$(ORD(S) EQ ORD(T)) = 0;

POSITIVE VARIABLE
 GAP(A,K)
 TFINISH;

NEGATIVE VARIABLE
 TSTART;

VARIABLE
 CVAL;

EQUATIONS
  OBJ
  BEGIN(A)
  END(A)
  BETWEEN(A,K,S)
  ONCE(S)
  ONCE1(S)
  TLIMIT
  START(A,K)
  FINISH(A,K)
  ;

*                     > > >  OBJECTIVE FUNCTION < < <
OBJ..
 CVAL =E= SUM((A,SS), VAL(SS)*V(A,SS));

*                     > > > subject to< < <
BEGIN(A)..
 SUM((FT,BB,TT), X(A,FT,BB,TT)) =E= 1;

END(A)..
 SUM((KT,SS,EE), X(A,KT,SS,EE)) =E= 1;

BETWEEN(A,K,SS)$(ORD(K) GT 1)..
 SUM(BB,X(A,K-1,BB,SS))$(ORD(K) EQ 2) +
 SUM(TT,X(A,K-1,TT,SS))$(ORD(K) GT 2)    =E=
 SUM(ST,X(A,K,SS,ST))  $(ORD(K) LT CARD(K)) +
 SUM(EE,X(A,K,SS,EE))  $(ORD(K) EQ CARD(K))   ;
```

```
ONCE(SS)..
 SUM(A,SUM((FT,BB),X(A,FT,BB,SS)) +
 SUM((KK,TT),X(A,KK,TT,SS)))     =E= SUM(A,V(A,SS));

ONCE1(SS)..
 SUM(A,V(A,SS)) =L= 1;

TLIMIT..
 AVAIL  =G= TFINISH - TSTART ;

START(A,K)$(ORD(K) LT CARD(K))..
 TSTART =L= (-SUM((FT,BB,TT),
               X(A,FT,BB,TT)*DIST(TT,BB))+GAP(A,K))$(ORD(K) EQ 1)
        + (SUM((FT,BB,TT),
               X(A,FT,BB,TT)*UTIME(TT)) + GAP(A,'1')
        +  SUM(L$(ORD(L) GT 1 AND ORD(L) LT ORD(K)),
               SUM((SS,TT),
                   X(A,L,SS,TT)*UTIME(TT))
              + GAP(A,L))
        - SUM((SS,TT,BB),
               X(A,K,SS,TT)*DIST(TT,BB))+GAP(A,K))$(ORD(K) GT 1);

FINISH(A,K)$(ORD(K) LT CARD(K))..
 TFINISH =G= (SUM((FT,BB,TT),
               X(A,FT,BB,TT)*(DIST(BB,TT)+UTIME(TT)))
              + GAP(A,K))$(ORD(K) EQ 1)
        + (SUM((FT,BB,TT),
               X(A,FT,BB,TT)*UTIME(TT)) + GAP(A,'1')
        +  SUM(L$(ORD(L) GT 1 AND ORD(L) LT ORD(K)),
               SUM((SS,TT),
                   X(A,L,SS,TT)*UTIME(TT))
              + GAP(A,L))
        +  SUM((SS,TT,EE),
               X(A,K,SS,TT)*(DIST(EE,TT)+UTIME(TT)))+GAP(A,K))
         $(ORD(K) GT 1);

*----------------------------------------------------------------
MODEL UNREP  / ALL /;
*UNREP.OPTFILE = 1;
*UNREP.CHEAT = 0.1;
SOLVE UNREP USING  MIP MAXIMIZING CVAL;

*-------------------Solution Report----------------------------
```

```
PARAMETER REPORT(A,*,K);
 REPORT(A,'LEAVE',K)$(ORD(K) LT CARD(K))
     =(- SUM((FT,BB,TT),
           X.L(A,FT,BB,TT)*DIST(TT,BB))+GAP.L(A,K))$(ORD(K) EQ 1)
      + (SUM((FT,BB,TT),
           X.L(A,FT,BB,TT)*UTIME(TT)) + GAP.L(A,'1')
      +  SUM(L$(ORD(L) GT 1 AND ORD(L) LT  ORD(K)),
           SUM((SS,TT),
              X.L(A,L,SS,TT)*UTIME(TT))
           + GAP.L(A,L))
     - SUM((SS,TT,BB),
           X.L(A,K,SS,TT)*DIST(TT,BB))+GAP.L(A,K))$(ORD(K) GT 1);

 REPORT(A,'ARRIVE',K)$(ORD(K) LT CARD(K) AND ORD(K) GT 1)
     = REPORT(A,'LEAVE',K) +
      SUM((S,T,BB),(DIST(T,BB)+GAP.L(A,K))*X.L(A,K,S,T)) ;

 REPORT(A,'ARRIVE',K)$(ORD(K) EQ 1)
     = REPORT(A,'LEAVE',K) +
      SUM((S,T,BB),DIST(T,BB)*X.L(A,K,S,T)) ;

 REPORT(A,'DEPART',K)$(ORD(K) LT CARD(K))
     = REPORT(A,'ARRIVE',K) +
      SUM((S,T),UTIME(T)*X.L(A,K,S,T)) ;

 REPORT(A,'RETURN',K)$(ORD(K) LT CARD(K))
     = (SUM((FT,BB,TT),
          X.L(A,FT,BB,TT)*(DIST(BB,TT)+UTIME(TT)))
          + GAP.L(A,K))$(ORD(K) EQ 1)
    + (SUM((FT,BB,TT),
          X.L(A,FT,BB,TT)*UTIME(TT)) + GAP.L(A,'1')
    +  SUM(L$(ORD(L) GT 1 AND ORD(L) LT  ORD(K)),
          SUM((SS,TT),
             X.L(A,L,SS,TT)*UTIME(TT))
          + GAP.L(A,L))
    +  SUM((SS,TT,EE),
         X.L(A,K,SS,TT)*(DIST(EE,TT)+UTIME(TT)))+GAP.L(A,K))
         $(ORD(K) GT 1);

OPTION X:1:2:1;
OPTION REPORT:3:1:1;
DISPLAY CVAL.L, DIST, AVAIL, X.L;
DISPLAY TSTART.L, TFINISH.L, REPORT, GAP.L;
```

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